Physics 115B, Spring 2022, Midterm Exam

Common notation: Let $|l, m\rangle$ represent an eigenstate of angular momentum $L^2 = l(l+1)\hbar^2$ and $L_z = m\hbar$. In the case of spin states, simply replace $L \to S$, $l \to s$.

Please put a "box" around each of your final answers.

1 Problem 1 - each question 2 points

Answer the following quick questions. No explanation needed unless the question starts with "Explain".
(a) Explain why the operator $\hat{r} \cdot \vec{p}$ is not hermitian, and therefore not a good quantum mechanical representation of the radial momentum operator.
(b) Excluding the spin degree of freedom, what is the degeneracy of the $n = 3$ energy level of the hydrogen atom?
(c) The spin of an electron is associated with the rotation of the electron around its axis. True or False?
(d) Consider the 2D vector space with basis vector the two eigenstates of S_z with $s = 1/2$: $|u_1\rangle = |1/2, 1/2\rangle$ and $|u_2\rangle = |1/2, -1/2\rangle$. Write the S_z^2 operator in matrix form.

2 Problem 2

Consider the following state with $l = 1$:

$$|\psi\rangle = a\,|1,1\rangle + b\,|1,0\rangle + c\,|1,-1\rangle$$

(a) Find a, b, c such the $|\psi\rangle$ is a properly normlized eigenstate of L_x with eigenvalue 0. For simplicity, take a, b, c real. There will be a sign ambiguity, *i.e.*, if $|\psi\rangle$ is an eigenstate, so is (-1) times $|\psi\rangle$, do not worry about that (6 points).

(b) Find the eigenstates of L_x with eigenvalues $\pm\hbar$ (4 points).

3 Problem 3

Consider a spherical infinite potential well, $V(r) = 0$ for $r < a$, $V(r) = \infty$ for $r > a$.

(a) Show that $\psi(r) = e^{ikr}/r$ is a solution of the radial equation for $l = 0$. Find the relation between k and the energy E (5 points).

(b) Find the allowed values of E. You can discard solutions where $\psi(r) \to \infty$ at $r = 0$ (5 points).

4 Question 4

Suppose that a particle is in the state $|l, m\rangle$.

(a) Find the expectation value of the x-component of angular momentum L_x (5 points).

(b) Find the expectation value of L_x^2 (5 points).