1. The initial state $|i\rangle = |pd\rangle$ has $I = I_3 = \frac{1}{2}$. Then the final states are either
   $|f\rangle = |\pi^0\ He^3\rangle = |1,0\rangle|\frac{1}{2},\frac{1}{2}\rangle = \sqrt{\frac{2}{3}}|\frac{3}{2},\frac{1}{2}\rangle - \sqrt{\frac{1}{3}}|\frac{1}{2},\frac{1}{2}\rangle$
   or
   $|f\rangle = |\pi^+\ H^3\rangle = |1,1\rangle|\frac{1}{2},-\frac{1}{2}\rangle = \sqrt{\frac{1}{3}}|\frac{3}{2},\frac{1}{2}\rangle + \sqrt{\frac{2}{3}}|\frac{1}{2},\frac{1}{2}\rangle$
   Since $I = \frac{1}{2}$ for the initial state, only the $I = \frac{1}{2}$ pieces of the final states contribute to $A_{i\to f}$. Thus, for the amplitudes $A(pd \to \pi^+ H^3) = \sqrt{2}A(pd \to \pi^0 H^3)$ and for the cross-sections $\sigma(pd \to \pi^+ H^3) = 2\sigma(pd \to \pi^0 H^3)$

2. (a) The commutator is given by

$$[H, T(a)] = \left[\frac{p^2}{2m} + V(x), e^{-iap/\hbar}\right] = \left[V(x), e^{-iap/\hbar}\right],$$

   i.e., only the potential part of the Hamiltonian matters since $T(a)$ can be written with momentum the only operator appearing. One can evaluate this by writing a series expansion for the exponential, writing the momentum operator as a derivative with respect to x, and expanding as a sum of commutators and operators; however, this is much more algebra than we would like to do. Instead, we will use the definition of the translation operator as translating a function:

$$\begin{aligned}[V(x), T(a)]\psi(x) &= V(x)\,[T(a)\psi(x)] - T(a)\,[V(x)\psi(x)] \\ &= V(x)\psi(x-a) - V(x-a)\psi(x-a) \\ &= V(x)T(a)\psi(x) - V(x-a)T(a)\psi(x) \\ &= (V(x) - V(x-a))\,T(a)\psi(x),\end{aligned}$$

   so $[H, T(a)] = [V(x), T(a)] = (V(x) - V(x-a))\,T(a)$.

   (b) We know p is the generator of spatial translations. We thus have

$$\frac{d\,\langle T(\varepsilon)\rangle}{dt} = \frac{d\,\langle I - i\varepsilon G/\hbar\rangle}{dt} = \frac{d\,\langle I\rangle}{dt} - \frac{i\varepsilon}{\hbar}\frac{d\,\langle p\rangle}{dt} = -\frac{i\varepsilon}{\hbar}\frac{d\,\langle p\rangle}{dt}$$

   and

$$[H, T(e)] = [\frac{p^2}{2m} + V(x), I - \frac{i\varepsilon p}{\hbar}] = -\frac{i\varepsilon}{\hbar}[\frac{p^2}{2m} + V(x), p] = -\frac{i\varepsilon}{\hbar}[V(x), p] = \varepsilon V'(x).$$

   Note that this is consistent of taking the infinitesimal limit of the expression in (a); the difference in the potential between two points goes to V' time the distance between the points, while the translation operator goes to the identity (since we only need it to lowest order). Ehrenfest's theorem thus gives

$$\frac{d\,\langle p\rangle}{dt} = -\,\langle V'(x)\rangle$$

   (c) We can write the general solution to the differential equation above as

$$\langle p\rangle_f = \langle p\rangle_i - \int_{t_i}^{t_f} \langle V'(x)\rangle\, dt.$$

   If the Hamiltonian is translationally-invariant, the potential does not vary in space, so $V'(x) = 0$ and $\langle p\rangle$ is constant.