1. The initial state $|i\rangle = |pd\rangle$ has $I = I_3 = \frac{1}{2}$. Then the final states are either

$|f\rangle = |\pi^0 H e^3\rangle = |1, 0\rangle |\frac{1}{2}, \frac{1}{2}\rangle = \sqrt{\frac{2}{3}} |\frac{3}{2}, \frac{1}{2}\rangle - \sqrt{\frac{1}{3}} |\frac{1}{2}, \frac{1}{2}\rangle$

or

$|f\rangle = |\pi^+ H^3\rangle = |1, 1\rangle |\frac{1}{2}, -\frac{1}{2}\rangle = \sqrt{\frac{1}{3}} |\frac{3}{2}, \frac{1}{2}\rangle + \sqrt{\frac{2}{3}} |\frac{1}{2}, \frac{1}{2}\rangle$

Since $I = \frac{1}{2}$ for the initial state, only the $I = \frac{1}{2}$ pieces of the final states contribute to $A_{i \rightarrow f}$. Thus, for the amplitudes $A(pd \rightarrow \pi^+ H^3) = \sqrt{2} \sigma(pd \rightarrow \pi^0 H^3)$ and for the cross-sections $\sigma(pd \rightarrow \pi^+ H^3) = 2 \sigma(pd \rightarrow \pi^0 H^3)$

2. (a) The commutator is given by

$$[H, T(a)] = \left[ \frac{p^2}{2m} + V(x), e^{-iap/h} \right] = \left[ V(x), e^{-iap/h} \right],$$

i.e., only the potential part of the Hamiltonian matters since $T(a)$ can be written with momentum the only operator appearing. One can evaluate this by writing a series expansion for the exponential, writing the momentum operator as a derivative with respect to $x$, and expanding as a sum of commutators and operators; however, this is much more algebra than we would like to do. Instead, we will use the definition of the translation operator as translating a function:

$$[V(x), T(a)]\psi(x) = V(x) [T(a)\psi(x)] - T(a) [V(x)\psi(x)]$$

$$= V(x)\psi(x - a) - V(x - a)\psi(x)$$

$$= V(x)T(a)\psi(x) - V(x - a)T(a)\psi(x)$$

$$= (V(x) - V(x - a)) T(a)\psi(x),$$

so $[H, T(a)] = [V(x), T(a)] = (V(x) - V(x - a)) T(a)$.

(b) We know $p$ is the generator of spatial translations. We thus have

$$\frac{d}{dt} \langle T(\varepsilon) \rangle = \frac{d}{dt} \langle I - i\varepsilon G/h \rangle = \frac{d}{dt} \langle I \rangle - \frac{i\varepsilon}{h} \frac{d}{dt} \langle p \rangle = -\frac{i\varepsilon}{h} \frac{d}{dt} \langle p \rangle$$

and

$$[H, T(\varepsilon)] = \left[ \frac{p^2}{2m} + V(x), I - \frac{i\varepsilon p}{h} \right] = -\frac{i\varepsilon}{h} \left[ \frac{p^2}{2m} + V(x), p \right] = -\frac{i\varepsilon}{h} [V(x), p] = \varepsilon V'(x).$$

Note that this is consistent of taking the infinitesimal limit of the expression in (a); the difference in the potential between two points goes to $V'$ time the distance between the points, while the translation operator goes to the identity (since we only need it to lowest order). Ehrenfest’s theorem thus gives

$$\frac{d}{dt} \langle p \rangle = -\langle V'(x) \rangle$$

(c) We can write the general solution to the differential equation above as

$$\langle p \rangle_f = \langle p \rangle_i - \int_{t_i}^{t_f} \langle V'(x) \rangle dt.$$