Physics 115B, Mastery Questions for Section 9 Spring 22

.

1. (Much of this introductory text is taken from Homework 8 Problem 4).
   The concept of isospin was introduced in the 1930s. As far as the strong interaction is concerned, protons and neutrons are the same. They are both spin $\frac{1}{2}$ fermions, they have (almost) the same mass, and the same strong interaction properties. So, ignoring E&M effects, they can be thought of as the same particle (a "nucleon") with an additional intrinsic quantum number called "isospin" (I) which has the same algebraic properties as spin or angular moment.
   The nucleon has $I = \frac{1}{2}$, so the proton and the neutron are an isospin doublet distinguished by the 3rd component of I, i.e., $I_3 = +\frac{1}{2}$ for the proton and $I_3 = -\frac{1}{2}$ for the neutron. We can write the states as
   $|p\rangle = |\frac{1}{2} \quad +\frac{1}{2}\rangle_N$ and
   $|n\rangle = |\frac{1}{2} \quad -\frac{1}{2}\rangle_N$
   where the notation is the same as the one we used for angular momentum, and the subscript N (which we can also drop, as long as we know what we are talking about) indicates that these are nucleon states.
   The deuteron d is a pn bound state with $I = 0$. Tritium $H^3 = pnn$ and $He^3 = ppn$ also form an isospin doublet (ie: $I = \frac{1}{2}$) with $I_3 = -\frac{1}{2}$ and $I_3 = +\frac{1}{2}$, respectively. Also: the three pions π^+ π^0 π^- make up an isospin triplet, (ie: $I = 1$) with third component $I_3 = 1, 0, -1$, respectively.
   You will learn in 115C that for $i \to f$ the probability of the process is given by the square of an amplitude $A_{i\to f} = \langle f|S|i\rangle$, where S here is the so-called S-matrix operator. In strong interactions, isospin (both I and I_3) are conserved, $A_{i\to f}$ does depend on I but not on I_3. Find ratio of the cross-sections (ie, the ratio of probabilities) for $pd \to \pi^0 He^3$ and $pd \to \pi^+ H^3$.

2. Recall Ehrenfest's theorem for time-independent operators:

$$\frac{d}{dt}\langle\mathcal{O}\rangle = \frac{i}{\hbar}\langle[H, \mathcal{O}]\rangle.$$

   (a) Consider the general Hamiltonian

$$H = \frac{p^2}{2m} + V(x).$$

Recall that the spatial translation operator is given by

$$T(a) = \exp(-iap/\hbar).$$

What is the commutator $[H, T(a)]$?

(b) Now, consider an infinitesimal translation $T(\epsilon) = I - \frac{i}{\hbar}\epsilon p$. Expanding your answer from (a) to first order in ϵ and using Ehrenfest's theorem, what can you conclude about $d\langle p\rangle/dt$?

(c) Now find an expression for $\langle p\rangle$. If our Hamiltonian is translationally-invariant, what can we conclude about $\langle p\rangle$?