1. (Much of this introductory text is taken from Homework 8 Problem 4).

The concept of isospin was introduced in the 1930s. As far as the strong interaction is concerned, protons and neutrons are the same. They are both spin $\frac{1}{2}$ fermions, they have (almost) the same mass, and the same strong interaction properties. So, ignoring E&M effects, they can be thought of as the same particle (a "nucleon") with an additional intrinsic quantum number called "isospin" ($I$) which has the same algebraic properties as spin or angular moment.

The nucleon has $I = \frac{1}{2}$, so the proton and the neutron are an isospin doublet distinguished by the 3rd component of $I$, i.e., $I_3 = +\frac{1}{2}$ for the proton and $I_3 = -\frac{1}{2}$ for the neutron. We can write the states as

$$|p\rangle = |\frac{1}{2} + \frac{1}{2}\rangle_N \quad \text{and} \quad |n\rangle = |\frac{1}{2} - \frac{1}{2}\rangle_N$$

where the notation is the same as the one we used for angular momentum, and the subscript $N$ (which we can also drop, as long as we know what we are talking about) indicates that these are nucleon states.

The deuteron $d$ is a $pn$ bound state with $I = 0$. Tritium $H^3 = pnn$ and $He^3 = ppm$ also form an isospin doublet (ie: $I = \frac{1}{2}$) with $I_3 = -\frac{1}{2}$ and $I_3 = +\frac{1}{2}$, respectively. Also: the three pions $\pi^+ \pi^0 \pi^-$ make up an isospin triplet, (ie: $I = 1$) with third component $I_3 = 1, 0, -1$, respectively.

You will learn in 115C that for $i \to f$ the probability of the process is given by the square of an amplitude $A_{i\to f} = (f|S|i)$, where $S$ here is the so-called $S$-matrix operator. In strong interactions, isospin (both $I$ and $I_3$) are conserved, $A_{i\to f}$ does depend on $I$ but not on $I_3$. Find ratio of the cross-sections (ie, the ratio of probabilities) for $pd \to \pi^0 He^3$ and $pd \to \pi^+ He^3$.

2. Recall Ehrenfest’s theorem for time-independent operators:

$$\frac{d}{dt} \langle O \rangle = i \frac{\hbar}{\hbar} \langle [H, O] \rangle$$

(a) Consider the general Hamiltonian
\[ H = \frac{p^2}{2m} + V(x). \]

Recall that the spatial translation operator is given by

\[ T(a) = \exp(-iap/h). \]

What is the commutator \([H, T(a)]\)?

(b) Now, consider an infinitesimal translation \(T(\epsilon) = I - \frac{i}{\hbar} \epsilon p\). Expanding your answer from (a) to first order in \(\epsilon\) and using Ehrenfest’s theorem, what can you conclude about \(d\langle p\rangle/dt\)?

(c) Now find an expression for \(\langle p\rangle\). If our Hamiltonian is translationally-invariant, what can we conclude about \(\langle p\rangle\)?