1. (a) The ground state is the state of lowest energy. The lowest energy two-particle wavefunction will have both particles in the lowest energy single particle state, so we have

$$\psi_0(\mathbf{r}_1, \mathbf{r}_2) = \psi_{100}(\mathbf{r}_1)\psi_{100}(\mathbf{r}_2)$$

(b) The spatial wavefunction above is symmetric. The spin state must therefore be antisymmetric, for which the only possibility is the singlet state $|0,0\rangle$.

(c) An electron falling from the first excited state $n = 2$ to the ground state releases

$$-4(13.6)\left(\frac{1}{4} - 1\right) = 40.8 \text{ eV}$$

of energy. The second electron needs only $-4(13.6)/4 = 13.6$ eV to be ionized, less than the energy released by the first electron. The second electron will escape with 27.2 eV.

(d) To have an antisymmetric total wavefunction, we will need either a symmetric or antisymmetric spatial wavefunction. The excited state spatial wavefunctions are thus given by

$$\psi_{nlm}(\mathbf{r}_1)\psi_{100}(\mathbf{r}_2) \pm \psi_{100}(\mathbf{r}_1)\psi_{nlm}(\mathbf{r}_2)$$

with $n > 1$, $0 \leq l < n$, $-l \leq m \leq l$.

(e) The symmetric spatial wavefunctions (plus sign) must have an antisymmetric spin state, so only the singlet $|0,0\rangle$ is alowed. The antisymmetric spatial wavefunctions (minus sign) must have a symmetric spin state, so all of the three triplet states $|1,1\rangle, |1,0\rangle, |1,-1\rangle$ are allowed.

(f) The ground state has no degeneracy, since the only allowed spin state is the singlet. The first excited, spatially symmetric state has $n = 2$, so $l = 0, 1$ are both allowed, as are all of the possible m values. The only allowed spin state is the singlet. There is thus $1 + 3 = 4$-fold degeneracy. For the first excited, spatially antisymmetric state, we have the same allowed l, m values, but now there are three possible spin states for each. There is thus $3(1 + 3) = 12$-fold degeneracy.