Physics 115B, Mastery Questions for Section 8 Spring 22

In today's section, we will get more practice with identical particles and the exchange interaction.

1. In this question, you will construct the electron states of helium. Recall that helium has 2 protons and 2 electrons. Ignore Coulomb repulsion between the electrons. In this approximation, the problem becomes very similar to that of hydrogen.

   (a) Construct the helium ground state spatial wavefunction $\psi_0(\mathbf{r_1}, \mathbf{r_2})$ in terms of hydrogenic wavefunctions $\psi_{n,l,m}(\mathbf{r})$. By 'hydrogenic' we just mean that the Bohr radius is adjusted to account for the fact that the nucleus charge is twice that of hydrogen.

   (b) What spin state must the ground state electrons be in to satisfy symmetrization requirements?

   (c) Now consider the case where the electrons occupy excited energy levels n and m, with $n, m > 1$. For any given electron in a helium atom, $E_n = -4(13.6\,\text{eV})/n^2$. If one electron falls back to the ground state, with all of the released energy being absorbed by the other electron, show that this other electron will become ionized ($E > 0$). With what energy does it escape the helium atom?

   (d) Since we are interested in *neutral* helium, we can therefore have only one electron in an excited n, l, m state, with the other in the ground state. Given this, construct the helium excited state spatial wavefunctions in terms of hydrogenic wavefunctions. Recall that the antisymmetry requirement applies to the total wavefunction.

   (e) What are the allowed spin states for each spatial wavefunction you constructed in (d)?

   (f) What is the degeneracy of the

      i. ground state?

      ii. first excited, spatially symmetric, state?

      iii. first excited, spatially antisymmetric, state?