Physics 115B, Mastery Questions for Section 6 Spring 22

1. Suppose we have a system comprising a graviton (spin-2) and an electron (spin-1/2).

   (a) If the individual spins have z-components $m_1 = 1$ and $m_2 = -1/2$, what is the z-component of the total system spin m? Is this a state of definite total spin, and if so what is it?

   (b) What total angular momenta are possible for the system when the orbital angular momentum is 0? What are the possible z-components of the total angular momenta, and what is the multiplicity of each?

   (c) What total angular momenta are possible for the system when the orbital angular momentum is 1? What are the possible z-components of the total angular momenta, and what is the multiplicity of each?

2. Recall that in classical electrodynamics, we want our theories to be gauge invariant, wherein the fields remain the same under some gauge transformation. We can write our fields $\mathbf{E}$ and $\mathbf{B}$ in terms of the vector potential $\mathbf{A}$ and scalar potential φ.

$$\mathbf{E} = -\nabla\varphi - \partial\mathbf{A}/\partial t,\ \mathbf{B} = \nabla \times \mathbf{A} \tag{1}$$

   (a) Check that these fields remain invariant under the gauge transformation

$$\varphi' \equiv \varphi - \frac{\partial\Lambda}{\partial t},\ \mathbf{A}' = \mathbf{A} + \nabla\Lambda \tag{2}$$

   where Λ is some real function of space and time.

   (b) Can you determine what Λ is? Why or why not?

3. A particles of spin 3/2, at rest in the laboratory, disintegrates into two particles, one of spin 1/2 and one of spin 0. What values are possible for the relative angular momemtum of the two particles? Note: the final state is a two body problem. A two body problem can be 'deconstructed" into the motion of the center of mass and the motion of a fictitious particle with reduced mass $\mu = m_1 m_2/(m_1 + m_2)$ about the center of mass. You have done this in Classical Mechanics, and we also did this in analyzing the Hydrogen atom. The relative angular momentum is the angular momentum of this fictious particle (See for example `http://tinyurl.com/y8ywv2mu`). In Classical Mechanics, for this problem, the relative (orbital) angular momentum in the final state is zero (why?), so such a decay could not happen (why?). But in Quantum Mechanics the relative angular momentum in a two-body decay does not have to be zero (!). An example of this process is the decay of the "delta" particle into a neutron and a pion: $\Delta^+(S = \frac{3}{2}) \rightarrow n(S = \frac{1}{2})\ \pi^+(S = 0)$.