Physics 115B, Mastery Questions for Section 3 Spring 22

Today we're going to briefly examine energy level transitions in hydrogen and then begin working with angular momentum.

1. Classically, both linear momentum and angular momentum are vectors. We generally label a momentum eigenstate by the momentum vector, $|\vec{p}\rangle$. Can we do this with angular momentum (i.e. write a state $\left|\vec{L}\right\rangle$), and if so, why don't we?

2. Consider a state $|l, m\rangle$ that is a simultaneous eigenstate of $\hat{L}^2$ and $\hat{L}_z$. Using raising and lowering operators, find $\langle L_x \rangle$ and $\langle L_y \rangle$. Recall that $\hat{L}_+ = \hat{L}_x + i\hat{L}_y$. How could you have found the answer without doing any calculation?

3. Consider a state $|l, m\rangle$ which is a simultaneous eigenfunction of $\hat{L}^2$ and $\hat{L}_z$. Recall

$$\hat{L}^2 |l, m\rangle = \hbar^2 l(l+1) |l, m\rangle$$
$$\hat{L}_z |l, m\rangle = \hbar m |l, m\rangle .$$

Additionally, as you will show on the problem set,

$$\hat{L}_+ |l, m\rangle = \hbar\sqrt{l(l+1) - m(m+1)}\, |l, m+1\rangle$$
$$\hat{L}_- |l, m\rangle = \hbar\sqrt{l(l+1) - m(m-1)}\, |l, m-1\rangle$$

For operators A and B, the standard deviation and generalized uncertainty principle are given by

$$\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$$
$$\Delta A \Delta B \geq \frac{1}{2} \left| \left\langle [A, B] \right\rangle \right| .$$

   (a) Find $\left\langle L^2 \right\rangle$ and $\left\langle L_z^2 \right\rangle$ for the cases $|l, 0\rangle$ and $|l, l\rangle$.
   (b) Find $\left\langle L_x^2 \right\rangle$ and $\left\langle L_y^2 \right\rangle$ for the same two cases.
   (c) Find ΔL_x and ΔL_y for both cases and discuss their significance.
   (d) Compare $\Delta L_x \Delta L_y$ with the uncertainty principle for these two cases.