

1. (a) Both position and momentum flip sign under parity;  $\Pi^{-1}x\Pi = -x$  and  $\Pi^{-1}p\Pi = -p$ .

(b) We have

$$\Pi^{-1}[x, p]\Pi = \Pi^{-1}xp\Pi - \Pi^{-1}px\Pi = \Pi^{-1}x\Pi\Pi^{-1}p\Pi - \Pi^{-1}p\Pi\Pi^{-1}x\Pi = (-x)(-p) - (-p)(-x) = xp - px = i\hbar$$

This matches  $\Pi^{-1}i\hbar\Pi = i\hbar$ ;  $i\hbar$  is a scalar and does not change under parity.

- (c) Angular momentum is position times momentum; since both of these flip sign under parity, angular momentum is unchanged.
- (d) Position is unaffected by time reversal. Since momentum is (proportional to) the derivative of position with respect to time, it should change sign under time reversal.
- (e) We have

$$\Theta^{-1}[x, p]\Theta = \Theta^{-1}xp\Theta - \Theta^{-1}px\Theta = \Theta^{-1}x\Theta\Theta^{-1}p\Theta - \Theta^{-1}p\Theta\Theta^{-1}x\Theta = x(-p) - (-p)x = -xp + px = -i\hbar$$

As prophesied, we must have  $\Theta^{-1}i\hbar\Theta = -i\hbar$  for everything to be consistent.

- (f) The complex conjugate of a complex conjugate takes one back to the original number;  $\Theta^2\psi = \Theta\Theta\psi = \Theta\psi^* = \psi^{**} = \psi$ .  $\Theta^2$  is thus the identity operator on bosons.
- (g) As stated earlier, position is unchanged under time reversal while linear momentum gets a minus sign. Angular momentum, which is position times momentum, thus changes sign under time reversal.
- (h) Applying  $\Theta$  twice, both terms are once again proportional to their initial state, and each has picked up a phase shift of  $\pi$ ; the second term picks it up on the first application of  $\Theta$  and the first term on the second application. Both  $a, b$  have been conjugated twice and thus returned to their original value, so acting with  $\Theta^2$  on a spin-1/2 state gives minus the original state.
2. (a) Since acceleration flips sign under parity, force must do so as well. To maintain the equality, each term on the right-hand-side must change sign. Charge is unchanged, so  $\mathbf{E}$  must change sign. Velocity flips sign under parity, so  $\mathbf{B}$  must remain unchanged.
- (b) Since acceleration is a second time derivative of position, it is unchanged under time reversal; each derivative flips sign but this cancels overall since there are an even number of them. Charge is unchanged, so  $\mathbf{E}$  must also be unchanged under time reversal. Velocity changes sign, so  $\mathbf{B}$  must also change sign for the second term to remain unchanged under time reversal.
- (c) The right-hand-side changes sign under charge conjugation, so the left-hand-side must as well. The derivative is unchanged; it has nothing to do with charge.  $\mathbf{E}$  therefore changes sign under charge conjugation.
- (d) Since  $\mathbf{E}$  changes sign, and all the derivative are unaffected by charge conjugation,  $\mathbf{B}$  must also change sign under charge conjugation.
- (e) Each of  $\mathbf{E}$  and  $\mathbf{B}$  flips sign under two of the transformations and is unchanged by the third. Both  $\mathbf{E}$  and  $\mathbf{B}$  are thus unchanged under the combined application of charge conjugation, parity, and time reversal.