Physics 115B, Problem Set 7

Due Friday, May 20, 5pm

Every problem is worth 10 points.
Every sub-question is worth the same, unless otherwise specified.

1 Three particles

Suppose you had three particles, one in state $\psi_a(x)$, one in state $\psi_b(x)$, and one in state $\psi_c(x)$. Assuming ψ_a, ψ_b, ψ_c are all orthonormal, construct the three-particle states representing

(a) distinguishable particles (2 points).

(b) identical bosons (4 points).

(c) identical fermions (4 points).

Keep in mind that (b) must be completely symmetric, under interchange of *any* pair of particles, and (c) must be completely anti-symmetric, in the same sense. *Hint: google "Slater determinant"*

2 Particles in a SHO

Consider two non-interacting, identical particles of mass m moving in a one-dimensional simple harmonic oscillator potential of frequency ω. Determine the energies and wavefunctions of the ground state and the first excited state for

(a) two spin-1 bosons

(b) two spin-1/2 fermions

3 The ground state of Lithium

Ignoring electron-electron repulsion, construct the ground state of Lithium ($Z = 3$). Start with a spatial wave function, remembering that only two electrons can occupy the hydrogenic ground state; the third goes to $\psi_{2,0,0}$. What is the energy of this state? Now tack on the spin, and antisymmetrize. What's the degeneracy of the ground state?

4 Silver

The density of silver is 10.49 g/cm^3, and its atomic weight is 107.9 g/mole.

(a) Calculate the Fermi energy E_F for silver. Assume $d = 1$, where d is defined on pg 218 of Griffiths, and give your answer in electron volts (eV).

(b) What is the corresponding electron velocity? *Hint: set* $E_F = \frac{1}{2}mv^2$. Is it safe to assume the electrons in silver are nonrelativistic?

(c) At what temperature T_F would the characteristic thermal energy ($k_B T$, where k_B is the Boltzmann constant and T is the temperature in Kelvin) equal the Fermi energy E_F for silver? Note: This is called the Fermi termperature; as long as the actual temperature is significantly below the Fermi temperature, the material can be regarded as "cold" in the sense that most of the electrons are in the lowest accessible state. Since the melting point of silver is 1235 K, solid silver is always cold.

(d) Calculate the degeneracy pressure of silver, in the free electron gas model.

5 Particle physics: meson decay.

The ρ^0 meson has spin 1 and the π^0 meson has spin 0. Show that the decay $\rho^0 \to \pi^0\pi^0$ is impossible.

PS: Both the π^0 and ρ^0 meson are maade up of quark-antiquark pairs, combination of $u\bar{u}$ and $d\bar{d}$, where u is the up-quark, d is the down-quark, and the bar on top is the symbol for antiquarks. In some sense, the ρ^0 is an excited state of the π^0. A "meson" is particle physics jargon for a bound state of a quark and an antiquark. A 'baryon" is particle physics jargon for a bound state of three quarks.
Note: quarks carry "color charge" red, blue, or green (R, B, or G). In the strong interaction color charge is the moral-equivalent of electric charge in the electromagnetic interaction, but while in E&M there is only one type of charge (electric), in the strong ineratction there are three types called colors. (They are not really colors, they are are just whimsical labels).
Just as atoms are electrical neutral, bound states of quarks and antiquarks must be color neutral. A quark-antiquark pair $q\bar{q}$ can be color neutral, e.g., $R\bar{R}$. Three quark qqq or antiquark $\bar{q}\bar{q}\bar{q}$ states can also be color neutral (RGB or $\bar{R}\bar{B}\bar{G}$). Protons are uud, neutrons are udd, antiprotons are $\bar{u}\bar{u}\bar{d}$, etc. It is also possible to have neutral $qqq\bar{q}\bar{q}$ ("pentaquark") states. These have (maybe) recently seen.