PHYSICS 115 B
HOMEWORK 5

page 1

(1) (a) $\ell_1 \otimes \ell_2 = \ell_1+\ell_2 \oplus \ell_1+\ell_2-1 \oplus \cdots \oplus |\ell_1-\ell_2|$

$$\Rightarrow \boxed{\ell = \frac{7}{2}, \frac{5}{2}, \frac{3}{2}, \frac{1}{2}}$$

(b) From the tables:

$$\boxed{\begin{aligned} \left|\tfrac{5}{2}\ \tfrac{1}{2}\right\rangle = {} & \sqrt{\tfrac{6}{35}}\,|2\ 2\rangle\left|\tfrac{3}{2}\ {-\tfrac{3}{2}}\right\rangle + \sqrt{\tfrac{5}{14}}\,|2\ 1\rangle\left|\tfrac{3}{2}\ {-\tfrac{1}{2}}\right\rangle + \\ & - \sqrt{\tfrac{3}{35}}\,|2\ 0\rangle\left|\tfrac{3}{2}\ \tfrac{1}{2}\right\rangle - \sqrt{\tfrac{27}{70}}\,|2\ {-1}\rangle\left|\tfrac{3}{2}\ \tfrac{3}{2}\right\rangle \end{aligned}}$$

(c)

$$\boxed{\begin{aligned} |2\ {-2}\rangle\left|\tfrac{3}{2}\ \tfrac{3}{2}\right\rangle = {} & \sqrt{\tfrac{1}{35}}\left|\tfrac{7}{2}\ {-\tfrac{1}{2}}\right\rangle - \sqrt{\tfrac{6}{35}}\left|\tfrac{5}{2}\ {-\tfrac{1}{2}}\right\rangle + \\ & + \sqrt{\tfrac{2}{5}}\left|\tfrac{3}{2}\ {-\tfrac{1}{2}}\right\rangle - \sqrt{\tfrac{2}{5}}\left|\tfrac{1}{2}\ {-\tfrac{1}{2}}\right\rangle \end{aligned}}$$

(2) (a) $\boxed{\tfrac{1}{2} \otimes \tfrac{1}{2} = 0 \oplus 1}$

(b) Start with 0 and 1, add $\frac{1}{2}$

$$0 \otimes \frac{1}{2} = \frac{1}{2}$$

$$1 \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{3}{2} \implies \boxed{\frac{1}{2} \text{ or } \frac{3}{2}}$$

(c) Pentaquark $\approx$ Meson + Baryon

Meson $S = 0, 1$

Baryon $S = 1/2, 3/2$

$$0 \otimes 1/2 = 1/2 \quad 0 \otimes 3/2 = 3/2 \quad 1 \otimes 1/2 = \frac{3}{2} \oplus \frac{1}{2}$$

$$1 \otimes 3/2 = \frac{5}{2} \oplus 3/2 \implies \boxed{\frac{1}{2} \text{ or } \frac{3}{2} \text{ or } 5/2}$$

(d) The splitting depends on the number of m states

Meson $s=0$, 1 beam
Meson $S=1$, 3 beams
Baryon $S=1/2$, 2 beams
Baryon $S=3/2$, 4 beams
Pentaquark $S=1/2$, 2 beams
Pentaquark $S=3/2$, 4 beams
Pentaquark $s=5/2$, 6 beams

③ $H = \varepsilon \vec{S}_1 \vec{S}_2$

$$S^2 = (S_1 + S_2)^2 = S_1^2 + S_2^2 + 2\vec{S}_1\vec{S}_2$$

$$\Rightarrow H = \frac{1}{2}\varepsilon\left[S^2 - S_1^2 - S_2^2\right]$$

$$\frac{1}{2} \otimes 1 = \frac{1}{2} \oplus \frac{3}{2}$$

So if $S = \frac{1}{2}$, eigenvalues of H are

$$E = \frac{1}{2}\varepsilon\left[\frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right) - 1(1+1)\right]\hbar^2$$

$$\boxed{E = -\varepsilon\hbar^2 \text{ for } S = 1/2 \text{, degeneracy } 2}$$

If $S = 3/2$ eigenvalues of H are

$$E = \frac{1}{2}\varepsilon\left[\frac{3}{2}\left(\frac{3}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right) - 1(1+1)\right]\hbar^2$$

$$E = \frac{1}{2}\varepsilon\left[\frac{15}{4} - \frac{3}{4} - 2\right]\hbar^2 = +\frac{1}{2}\varepsilon\hbar^2$$

$$\boxed{E = \frac{1}{2}\varepsilon\hbar^2 \text{ for } S = 3/2 \text{, degeneracy} = 4}$$

page 4

4

(a) $|1\ -1\rangle = \sqrt{\frac{3}{4}}\left|\frac{3}{2}\ -\frac{1}{2}\right\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle - \sqrt{\frac{1}{4}}\left|\frac{3}{2}\ -\frac{3}{2}\right\rangle\left|\frac{1}{2}\ +\frac{1}{2}\right\rangle$

For the spin $= \frac{3}{4}$ particle, a measurement of S_z will yield

$$\boxed{\begin{array}{l} S_z = -\frac{1}{2}\hbar,\ \text{probability} = \frac{3}{4} \\ S_z = -\frac{3}{2}\hbar,\ \text{probability} = \frac{1}{4} \end{array}}$$

(b) The measurement of (a) has collapsed the state as $\left|\frac{3}{2}\ -\frac{3}{2}\right\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$

From CG Tables

$$\left|\frac{3}{2}\ -\frac{3}{2}\right\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \sqrt{\frac{1}{4}}\left|2\ -\frac{1}{2}\right\rangle - \sqrt{\frac{3}{4}}\left|1\ -\frac{1}{2}\right\rangle$$

$$\boxed{\begin{array}{l} S^2 = 2(2+1)\hbar^2 = 6\hbar^2,\ \text{probability} = \frac{1}{4} \\ S^2 = 1(1+1)\hbar^2 = 2\hbar^2,\ \text{probability} = \frac{3}{4} \end{array}}$$

page 5

(c) Orbital $|2\ -1\rangle$

Spin $|\tfrac{1}{2}\ \tfrac{1}{2}\rangle$

$$|2\ -1\rangle|\tfrac{1}{2}\ \tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{5}}\,|\tfrac{5}{2}\ -\tfrac{1}{2}\rangle + \sqrt{\tfrac{3}{5}}\,|\tfrac{3}{2}\ -\tfrac{1}{2}\rangle$$

$J = L + S$

$$\boxed{\begin{aligned} J^2 &= \tfrac{5}{2}\left(\tfrac{5}{2}+1\right)\hbar^2 = \tfrac{35}{4}\hbar^2 \quad \text{probability} = \tfrac{2}{5} \\ J^2 &= \tfrac{3}{2}\left(\tfrac{3}{2}+1\right)\hbar^2 = \tfrac{15}{4}\hbar^2 \quad \text{probability} = \tfrac{3}{5} \end{aligned}}$$