Physics 115B, Problem Set 5

Due Friday, May 6, 5pm

Every problem is worth 10 points.
Every sub-question is worth the same, unless otherwise specified.

1 Clebsch-Gordan (CG) Gymnastics

Consider the addition of angular momenta 2 and $\frac{3}{2}$.

(a) The resulting states will be labelled $|\ell, m\rangle$. What are the possible values of ℓ. (4 points).

(b) One of the possible resulting states is $|l, m\rangle = \left|\frac{5}{2}, \frac{1}{2}\right\rangle$. Write this state as a linear combination of $|2, m_1\rangle \left|\frac{3}{2}, m_2\right\rangle$. (3 points).

(c) Write the state $|2, -2\rangle \left|\frac{3}{2}, \frac{3}{2}\right\rangle$ as a linear coombination of the $|\ell, m\rangle$'s. (3 points).

A link to a table of CG coefficients is available on the class website.

2 Mesons, Baryons, Pentaquarks

Quarks carry total spin $s = 1/2$. Three quarks bind together to make a *baryon* (such as a proton or a neutron), while two quarks (technically a quark and an antiquark, but the distinction is irrelevant here) bind together to make a *meson* (such as a pion or a kaon). Assume the quarks are in the ground state with zero orbital angular momentum.

(a) What spins are possible for mesons?

(b) What spins are possible for baryons?

(c) On July 13, 2015, the LHCb collaboration at CERN reported results consistent with the discovery of a *pentaquark* state, consisting of five quarks (technically four quarks and an antiquark). Assuming the quarks are in the ground state with zero orbital angular momentum, what spins are possible for the pentaquark state?

(d) You can produce separate beams of baryons, mesons, and pentaquarks in your laboratory and send them through a Stern-Gerlach (SG) apparatus. How many beams might you expect each of the samples to split into as it passes through the SG apparatus? **Note:** we did not cover SG in class; please read about it in Griffiths, Example 4.4.

3 Coupled Spins

Consider two particles. Particle 1 has spin $s_1 = 1/2$, while particle 2 has spin $s_2 = 1$. The particles are at rest and subject to an interaction that depends on their relative spin orientations, corresponding to the Hamiltonian

$$H = \epsilon \vec{S}_1 \cdot \vec{S}_2$$

Here $\vec{S}_1$ is the spin operator for particle 1 and $\vec{S}_2$ is the spin operator for particle 2. What are the energy eigenvalues of this Hamiltonian, and what are their degeneracies?

4 Clebsch-Gordan analysis

(a) A particle of spin $s_1 = 3/2$ and a particle of spin $s_2 = 1/2$ are at rest in a configuration where the total spin of the two particles is 1 and a measurement of the total spin of the two particles in the $\hat{z}$ direction yields $-\hbar$. If you then measured the spin of the spin-3/2 particle in the $\hat{z}$ direction, what values might you obtain, and with what probabilities? (4 points).

(b) Imagine that the outcome of your measurement in part (a) was $-3\hbar/2$. You subsequently measure the total spin of the two-particle system. What values might you obtain, and with what probabilities? (3 points).

(c) An electron with spin up in the $\hat{z}$ direction is in the state $\psi_{n,\ell,m} = \psi_{4,2,-1}$ of the hydrogen atom. If you measured the total angular momentum (orbital angular momentum plus spin) of the electron, what values might you obtain, and with what probabilities? (3 points).