Physics 115B, Problem Set 4

Due Sunday, May 1, 5 pm

1 Angular Eigenstates

Consider the eigenfunctions of the orbital angular momentum operators L^2 and L_z with $\ell = 1$, namely $|\ell, m\rangle = |1, -1\rangle, |1, 0\rangle, |1, 1\rangle$.

(a) Use the raising and lowering operators $L_\pm$ to determine the states $L_x |1, -1\rangle$, $L_x |1, 0\rangle$, and $L_x |1, 1\rangle$.

(b) Use your results from part (a) to find the $\ell = 1$ eigenstates and eigenvalues of the operator L_x in terms of the states $|1, -1\rangle, |1, 0\rangle, |1, 1\rangle$.

(c) Now consider representations of these states and operators in spherical coordinates, namely $|1, -1\rangle = Y_1^{-1}, |1, 0\rangle = Y_1^0, |1, 1\rangle = Y_1^1$ and

$$L_x = -i\hbar \left(-\sin\phi \frac{\partial}{\partial\theta} - \cos\phi \cot\theta \frac{\partial}{\partial\phi} \right)$$

Using these representations, verify that the states you found in part (b) are eigenstates of L_x by explicit computation.

2 Spin in $\hat{y}$

(a) Find the eigenvalues and eigenspinors of the spin operator S_y in the basis formed by eigenstates of S_z. Include a proper normalization for the eigenspinors.

(b) If you measured S_y on a particle in the general state

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix}$$

(in the basis formed by eigenstates of S_z), what values might you get, and with what probabilities? Here a and b can be complex numbers, and you can assume that the state is normalized, ie, $|a|^2 + |b|^2 = 1$.

(c) If you measured S_y^2 on a particle in the same state as part (b), what outcomes might you get, and with what probabilities?

3 An Electron Spin State

Consider an electron in the spin state

$$\chi = A \begin{pmatrix} 4i \\ 3 \end{pmatrix}$$

(a) Determine the normalization constant A.

(b) Find the expectation values of S_x, S_y, S_z, and S^2 in this state.

(c) Find the standard deviations $\sigma_{S_x}, \sigma_{S_y}, \sigma_{S_z}$ in this state.

4 Spin One

Construct a matrix representation of the operators S_x, S_y, S_z, and S^2 for a particle of spin 1, in the basis of eigenstates of S_z.