1 Angular Eigenstates

Consider the eigenfunctions of the orbital angular momentum operators $L^2$ and $L_z$ with $\ell = 1$, namely $|\ell, m\rangle = |1, -1\rangle, |1, 0\rangle, |1, 1\rangle$.

(a) Use the raising and lowering operators $L_{\pm}$ to determine the states $L_x |1, -1\rangle$, $L_x |1, 0\rangle$, and $L_x |1, 1\rangle$.

(b) Use your results from part (a) to find the $\ell = 1$ eigenstates and eigenvalues of the operator $L_x$ in terms of the states $|1, -1\rangle, |1, 0\rangle, |1, 1\rangle$.

(c) Now consider representations of these states and operators in spherical coordinates, namely $|1, -1\rangle = Y_{-1}^1$, $|1, 0\rangle = Y_0^0$, $|1, 1\rangle = Y_1^1$ and

$$L_x = -i\hbar \left( -\sin \phi \frac{\partial}{\partial \theta} - \cos \phi \cot \theta \frac{\partial}{\partial \phi} \right)$$

Using these representations, verify that the states you found in part (b) are eigenstates of $L_x$ by explicit computation.

2 Spin in $\hat{y}$

(a) Find the eigenvalues and eigenspinors of the spin operator $S_y$ in the basis formed by eigenstates of $S_z$. Include a proper normalization for the eigenspinors.

(b) If you measured $S_y$ on a particle in the general state

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix}$$

(in the basis formed by eigenstates of $S_z$), what values might you get, and with what probabilities? Here $a$ and $b$ can be complex numbers, and you can assume that the state is normalized, ie, $|a|^2 + |b|^2 = 1$.

(c) If you measured $S_y^2$ on a particle in the same state as part (b), what outcomes might you get, and with what probabilities?
3 An Electron Spin State

Consider an electron in the spin state

\[ \chi = A \begin{pmatrix} 4i \\ 3 \end{pmatrix} \]

(a) Determine the normalization constant \(A\).

(b) Find the expectation values of \(S_x, S_y, S_z\), and \(S^2\) in this state.

(c) Find the standard deviations \(\sigma_{S_x}, \sigma_{S_y}, \sigma_{S_z}\) in this state.

4 Spin One

Construct a matrix representation of the operators \(S_x, S_y, S_z\), and \(S^2\) for a particle of spin 1, in the basis of eigenstates of \(S_z\).