Physics 110B Practice Final Solutions

Problem 1

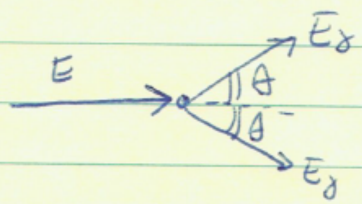
Working with c=1. In the end result, to restore the right units, if you care about that, change m into mc^2.

(a) $E_{INITIAL} = E + m$

$E_{FINAL} = 2E_\gamma$

CONSERVATION OF E $\Rightarrow$ $\boxed{E_\gamma = \frac{1}{2}(E+m)}$

(b)



Because there cannot be any vertical momentum in the final state, since the two photons have the same energy, the two angles must be the same

Conservation of $\vec{P}$: $\sqrt{E^2-m^2} = 2E_\gamma \cos\theta$ $\qquad \cos\theta = \frac{\sqrt{E^2-m^2}}{2E_\gamma}$

$\cos\theta = \frac{\sqrt{(E+m)(E-m)}}{E+m}$ $\qquad \boxed{\cos\theta = \sqrt{\frac{E-m}{E+m}}}$

Problem 2

(a) The induced current I_c must be such as to counteract ~~minimize~~ the change in flux. Since the solenoid field is to the right and is decreasing, I_c must be such that it makes a $\vec{B}$ field ~~to the~~ pointing to the right.
Using the right-hand-rule, this means I_c goes from LEFT TO RIGHT

(b) $$\Phi = 5 B \pi \left(\frac{d}{2}\right)^2 = \frac{5}{4}\pi d^2 B$$

5 turns of coil

$$B = \frac{4\pi I_s (N/L)}{c} \Rightarrow \Phi(t) = \frac{5\pi^2 N d^2 I_s(t)}{Lc}$$

induced emf

$$I_c = \frac{\mathcal{E}}{R} = \frac{1}{R}\left(-\frac{1}{c}\right)\frac{d\Phi}{dt} = -\frac{1}{Rc}\frac{d\Phi}{dt}$$

this sign is meaningless we have already figured out the direction....

$$I_c = -\frac{1}{Rc}\,\frac{5\pi^2 N d^2}{Lc}\,\frac{dI_s}{dt}.$$

But $I_s = I_0 e^{-t/\tau} \Rightarrow \frac{dI_s}{dt} = -\frac{1}{\tau} I_0 e^{-t/\tau}$

$$\boxed{I_c = \frac{5\pi^2 N d^2}{R\tau L c^2}\, e^{-t/\tau}}$$

Problem 3

(a) $J = \frac{I}{\pi R^2} = \frac{\alpha t}{\pi R^2}$

$E = \frac{J}{\sigma} = \frac{\alpha t}{\pi R^2 \sigma}$

Displacement current $J_D = \varepsilon_0 \frac{\partial E}{\partial t}$ $(r<R)$

$\boxed{J_D = \frac{\varepsilon_0 \alpha}{\pi R^2 \sigma}}$ Independent of r for $r<R$. Independent of t

(b) Maxwell eqtn $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$\vec{\nabla} \times \vec{B} = \mu_0 (\vec{J} + \vec{J}_D)$

When turning this into an integral equation this becomes $\int \vec{B}\, d\vec{\ell} = \mu_0 (I_{enclosed} + I_{D\_enclosed})$

Take path as circle of radius $r > R$

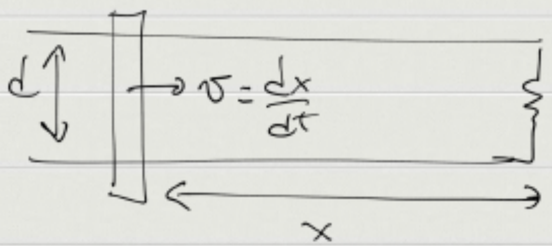
Also $I_{D\_enclosed} = J_D \cdot \pi R^2 = \frac{\varepsilon_0 \alpha}{\sigma}$

$B\, 2\pi r = \mu_0 \alpha t + \frac{\mu_0 \varepsilon_0 \alpha}{\sigma}$

$\boxed{B = \frac{\mu_0 \alpha}{2\pi r}\left[t + \frac{\varepsilon_0}{\sigma}\right]}$

Problem 4

(a) $\mathcal{E} = -\frac{d\Phi}{dt} \qquad I = \frac{\mathcal{E}}{R}$



$\Phi = x d B \qquad \frac{d\Phi}{dt} = dB\frac{dx}{dt} = dBv$

$\Rightarrow \mathcal{E} = -dBv \qquad \boxed{I = \frac{Bdv}{R}}$

where I dropped the minus sign

(b) $F = IBd = \frac{B^2 d^2 v}{R}$

(c) $\text{Power} = Fv = \frac{B^2 d^2 v^2}{R}$

Note also $\text{Power} = I^2 R = \frac{B^2 d^2 v^2}{R^2} R$

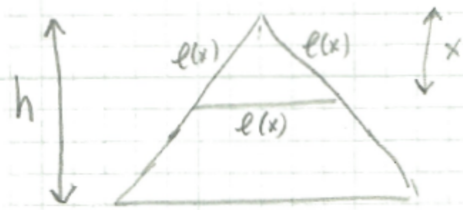
$\text{Power} = \frac{B^2 d^2 v^2}{R}$ ✓

Problem 5

All angles in the triangle are $60°$.

$\Rightarrow$ Height of triangle $h = a \sin 60° = \frac{\sqrt{3}}{2} a$

Picture of triangle



$x = \ell \sin 60°$

$\Rightarrow \ell = \frac{2}{\sqrt{3}} x$

$B(x) = \frac{\mu_0 I}{2\pi(x+b)}$ $(x+b$ is the distance from the wire$)$

Flux through a small horizontal strip at x : $d\phi = B(x)\, \ell\, dx$

But since $\ell = \frac{2}{\sqrt{3}} x$, $d\phi = \frac{2B(x)}{\sqrt{3}} x\, dx = \frac{\mu_0 I}{\sqrt{3}\pi} \frac{x}{x+b} dx$

Total flux $\phi = \frac{\mu_0 I}{\sqrt{3}\pi} \int_0^h \frac{x}{x+b} dx = \frac{\mu_0 I}{\sqrt{3}\pi} \int_0^h \frac{x+b-b}{x+b} dx$

$$\phi = \frac{\mu_0 I}{\sqrt{3}\pi} \int_0^h dx - \frac{\mu_0 I b}{\sqrt{3}\pi} \int_0^h \frac{dx}{x+b}$$

$$\phi = \frac{\mu_0 I}{\sqrt{3}\pi} h - \frac{\mu_0 I b}{\sqrt{3}\pi} \log \frac{h+b}{b}$$

And since $h = \frac{\sqrt{3}}{2} a$ $\phi = \frac{\mu_0 I}{\pi} \left[\frac{a}{2} - \frac{b}{\sqrt{3}} \log \frac{\frac{\sqrt{3}}{2} a + b}{b} \right]$

And finally $\boxed{M = \phi / I = \frac{\mu_0}{\pi} \left[\frac{a}{2} - \frac{b}{\sqrt{3}} \log \frac{\frac{\sqrt{3}}{2} a + b}{b} \right]}$

Problem 6

The speed of the wave is:

$$v = \frac{1}{\sqrt{\epsilon\mu_o}} = \frac{1}{\sqrt{f(x)\mu_o}}$$

But also

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} \Rightarrow \mathrm{d}x = v\,\mathrm{d}t \Rightarrow \mathrm{d}t = \frac{\mathrm{d}x}{v}$$

With the speed from the previous relationship,

$$\mathrm{d}t = \sqrt{\mu_o}\sqrt{f(x)}\,\mathrm{d}x$$

The time needed for the wave to pass the layer of thickness l is found by integration:

$$t = \sqrt{\mu_o}\int_0^l \sqrt{f(x)}\,\mathrm{d}x$$

Now let us obtain the constants A and b for function $f(x)$ from the boundary conditions.

$$f(0) = \epsilon_1, f(l) = \epsilon_2, \epsilon_2 < \epsilon_1$$
$$f(0) = A = \epsilon_1, f(l) = A\mathrm{e}^{-bl} = \epsilon_2$$

From the second equation it follows that $\epsilon_1\mathrm{e}^{-bl} = \epsilon_2$,

$$\mathrm{e}^{-bl} = \frac{\epsilon_2}{\epsilon_1} \Rightarrow -bl = \ln\frac{\epsilon_2}{\epsilon_1} \Rightarrow b = -\frac{1}{l}\ln\frac{\epsilon_2}{\epsilon_1}$$

The time is, therefore,

$$t = \sqrt{\mu_o}\int_0^l \sqrt{Ae^{-bx}}\,\mathrm{d}x = \sqrt{\mu_o A}\left(-\frac{2}{b}\mathrm{e}^{-\frac{bx}{2}}\right)\Bigg|_0^l = \frac{2}{b}\sqrt{\mu_o A}\left(1 - \mathrm{e}^{-\frac{bl}{2}}\right)$$

Back to the constants $A = \epsilon_1$, $b = -\frac{1}{l}\ln\frac{\epsilon_2}{\epsilon_1}$ and

$$t = \frac{2}{-\frac{1}{l}\ln\frac{\epsilon_2}{\epsilon_1}}\sqrt{\mu_o\epsilon_1}\left(1 - \mathrm{e}^{-\frac{1}{2}\ln\frac{\epsilon_2}{\epsilon_1}}\right) = \frac{2l\sqrt{\mu_o}}{\ln\frac{\epsilon_2}{\epsilon_1}}(\sqrt{\epsilon_2} - \sqrt{\epsilon_1}) = \frac{2l\sqrt{\mu_o}}{\ln\frac{\epsilon_1}{\epsilon_2}}(\sqrt{\epsilon_1} - \sqrt{\epsilon_2})$$

Problem 7

The motion of the electron involves energy that can be broken into three pieces: energy associated with moving a charged particle though a potential difference, kinetic energy, and energy to account for radiative losses

$$U_{\text{pot}} = qV \quad U_{\text{kin}} = \frac{1}{2}mv^2 \quad U_{\text{rad}} = P_{\text{rad}}t$$

We will consider $v \ll c$ so the Larmor formula can be used.

$$P_{\text{rad}} = \frac{\mu_o q^2 a^2}{6\pi c}$$

The initial kinetic energy is zero and the final is given by

$$U_{\text{kin}} = \frac{1}{2}m_e v^2 = \frac{1}{2}m_e(2ad) = m_e ad$$

where we use

$$v^2 = v_o^2 + 2ad$$

with $v_o = 0$. For the potential energy, $q = -e$ and $V = -V_o$, so

$$U_{\text{pot}} = (-e)(-V_o) = eV_o$$

For the radiation energy, we have

$$P_{\text{rad}} = \frac{\mu_o q^2 a^2}{6\pi c} = \frac{\mu_o e^2 a^2}{6\pi c}$$

and considering,

$$a = \frac{v}{t} \Rightarrow t = \frac{v}{a} = \frac{\sqrt{2ad}}{a} = \sqrt{\frac{2d}{a}}$$

we have

$$U_{\text{rad}} = P_{\text{rad}}t = \frac{\mu_o e^2 a^2}{6\pi c}\sqrt{\frac{2d}{a}} = \frac{\mu_o e^2 a^{3/2}}{3\pi c}\sqrt{\frac{d}{2}}$$

The total energy is then given by

$$U = U_{\text{kin}} + U_{\text{pot}} + U_{\text{rad}} = m_e ad + eV_o + \frac{\mu_o e^2 a^{3/2}}{3\pi c}\sqrt{\frac{d}{2}}$$

$$= ad\left(m_e + \frac{\mu_o e^2}{3\pi c}\sqrt{\frac{a}{2d}}\right) + eV_o$$