PHYSICS 110B FINAL

PROBLEM 1

Work with $c=1$

(a) In rest frame $P_{INIT} = (M, \vec{0})$

$$P_{FINAL} = \left(\sqrt{q^2+m^2} \quad \vec{q}\right) + \left(q \quad -\vec{q}\right)$$

electron

neutrino

Note: conservation of momentum is trivial and I just applied it above

Conservation of Energy $M = q + \sqrt{q^2+m^2}$

$$(M-q)^2 = q^2+m^2$$

$$M^2 + q^2 - 2Mq = q^2 + m^2$$

$$q = \frac{M^2 - m^2}{2M}$$ Restore factor of c: $\boxed{q = \frac{M^2 - m^2}{2M} c}$

(b) The boost is in the +ve x-direction

The β of the boost is the same as the β of the muon

$$p' = \gamma M \beta \Rightarrow \gamma\beta = p'/M$$

Also $\gamma = E'/M \Rightarrow \gamma = \sqrt{p'^2 + M^2}/M$

The momentum of the neutrino in the original frame was $-q$ in the x-direction. There is no y- or z-component anywhere

$$k' = \gamma(-q + \beta q)$$

$$k' = \frac{p' - \sqrt{p'^2 + M^2}}{M}\, q$$

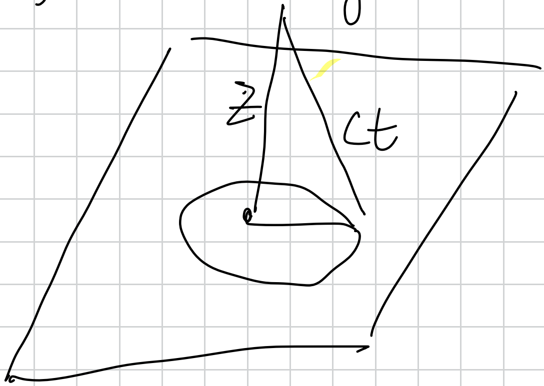
Restoring factors of c

$$\boxed{k' = \frac{p' - \sqrt{p'^2 + M^2 c^2}}{Mc}\, q}$$

PROBLEM 2

Since $\rho = 0 \Rightarrow V = 0$ always

For $\vec{A}$



only the circle of radius

$$r_0 = \sqrt{c^2t^2 - z^2}$$

contributes

$$\vec{A} = \frac{\mu_0 K \hat{x}}{4\pi} \int_0^{r_0} \int_0^{2\pi} \frac{r \, d\phi \, dr}{s}$$

where $s = \sqrt{z^2 + r^2}$

$$\vec{A} = \frac{\mu_0 K \hat{x}}{4\pi} 2\pi \int_0^{r_0} \frac{r}{\sqrt{z^2 + r^2}} dr$$

$$\vec{A} = \frac{\mu_0 K \hat{x}}{2} \left[\sqrt{z^2 + r^2} \right]_0^{r_0}$$

$$\vec{A} = \frac{\mu_0 K \hat{x}}{2} \left[\sqrt{z^2 + r_0^2} - z \right]$$

$$\vec{A} = \frac{\mu_0 k \hat{x}}{2}\left[\sqrt{z^2 + c^2t^2 - z^2} - z\right]$$

$$\vec{A} = \frac{\mu_0 k \hat{x}}{2}(ct - z)$$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t}$$

$$\boxed{\vec{E} = -\frac{\mu_0 c k \hat{x}}{2}}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\boxed{\vec{B} = -\frac{\mu_0 k}{2}\hat{y}}$$

PROBLEM 3

(a) $E = V/d = \frac{Q/C}{d} = \frac{Q}{d}\frac{d}{\varepsilon_0 \pi R^2} = \frac{1}{\varepsilon_0 \pi R^2} Q$

$$\frac{\partial E}{\partial t} = \frac{1}{\varepsilon_0 \pi R^2}\frac{\partial Q}{\partial t} = I/\varepsilon_0 \pi R^2$$

$\boxed{J_d = \frac{I}{\pi R^2} \text{ in the direction } \perp \text{ to the plates}}$

This same equation has been derived in Section 7.3.2 of Griffiths

(b) Using equation 7.39 of Griffiths

$$\oint \vec{B}\, d\vec{\ell} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t}\, d\vec{a}$$

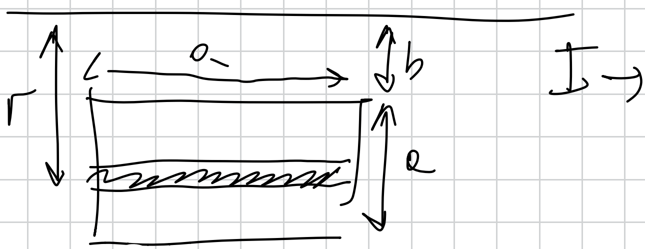
and choosing a circular path inside the capacitor, where $I_{enclosed} = 0$

$$B\, 2\pi r = \mu_0 \varepsilon_0\, \pi r^2 \frac{I}{\varepsilon_0 \pi R^2}$$

$$\boxed{B(r) = \frac{\mu_0 I r}{2\pi R^2}}$$

PROBLEM 4

We calculate the flux of $\vec{B}$ due to the wire through the square loop for a current I going through the wire. Then $M = \Phi / I$



$d\Phi$ = flux through shaded area $= B(r)\, a\, dr$

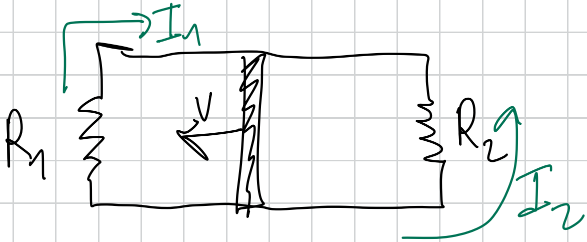
$$B(r) = \frac{\mu_0 I}{2\pi r} \qquad d\Phi = \frac{\mu_0 I a}{2\pi} \frac{dr}{r}$$

$$\Phi = \frac{\mu_0 I a}{2\pi} \int_b^{a+b} \frac{dr}{r} = \frac{\mu_0 I a}{2\pi} \log \frac{a+b}{a}$$

$$M = \Phi / I \qquad \boxed{M = \frac{\mu_0 a}{2\pi} \log \frac{a+b}{b}}$$

PROBLEM 5

(a)



(b) $\mathcal{E} = -\dfrac{d\Phi}{dt}$ $\quad |\mathcal{E}| = BLv$

$$\boxed{I_1 = \frac{\mathcal{E}}{R_1} = \frac{BLv}{R_1}} \qquad \boxed{I_2 = \frac{\mathcal{E}}{R_2} = \frac{BLv}{R_2}}$$

(c) $\text{Power} = I_1^2 R_1 + I_2^2 R_2$

$$\boxed{\text{Power} = B^2L^2v^2\left[\frac{1}{R_1} + \frac{1}{R_2}\right]}$$

(d) By conservation of energy

$\text{Power} = Fv$

$$\boxed{F = B^2L^2v\left[\frac{1}{R_1} + \frac{1}{R_2}\right]}$$

Alternatively, total current through rod is $I = I_1 + I_2$

Magnetic Force $F = ILB$

$$F = \left(\frac{BLv}{R_1} + \frac{BLv}{R_2}\right) LB$$

$$\boxed{F = B^2 L^2 v \left[\frac{1}{R_1} + \frac{1}{R_2}\right]}$$

Same answer!

(Note: the magnetic force must be balanced by the external force)

PROBLEM 6

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & 0 & 0 \end{vmatrix} = -kE_0\,\hat{y}\,\sin kz\cos\omega t$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \Rightarrow \boxed{\vec{B} = \frac{k}{\omega}E_0 \sin kz \sin\omega t\,\hat{y}}$$

I could go one step further, and from

$\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\dfrac{\partial\vec{E}}{\partial t}$ I could also derive that

$k/\omega = \dfrac{1}{c}$, but it is not necessary

Also, strictly speaking my solution up to this point could be

$$\vec{B} = \frac{k}{\omega}E_0 \sin kz \sin\omega t\,\hat{y} + \vec{V}$$

where $\vec{V}$ is a vector independent of time

From $\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ and from $\vec{\nabla} \vec{B} = 0$

I would get that $\vec{\nabla} \times \vec{V} = \vec{\nabla} \vec{V} = 0$

$\vec{V}$ = constant satisfies this condition but there are an infinite other possibilities

$$\boxed{\text{GRE QUESTIONS}}$$

7 : E

8 : C

9 : B