HOMEWORK 8

GRIFFITHS 11.1

Equation 11.12 $V(r,\theta,t) = \frac{P_0\cos\theta}{4\pi\varepsilon_0 r}\left[-\frac{\omega}{c}\sin\left(\omega t - \frac{\omega r}{c}\right) + \frac{1}{r}\cos\left(\omega t - \frac{\omega r}{c}\right)\right]$

Equation 11.17 $\vec{A}(r,\theta,t) = -\frac{\mu_0 P_0 \omega}{4\pi r}\sin\left(\omega t - \frac{\omega r}{c}\right)\left(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}\right)$

$$\vec{\nabla}\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\sin\theta A_\theta$$

$$\vec{\nabla}\vec{A} = \left[\frac{1}{r^2}\left(2rA_r + r^2\frac{\partial A_r}{\partial r}\right) + \frac{1}{r\sin\theta}\left(\cos\theta A_\theta + \sin\theta\frac{\partial A_\theta}{\partial\theta}\right)\right]$$

$$\vec{\nabla}\vec{A} = -\frac{\mu_0 P_0\omega}{4\pi}\left[\frac{2}{r^2}\cos\theta\sin\left(\omega t - \frac{\omega r}{c}\right) + \frac{\cos\theta}{r}\left(-\frac{\omega}{c}\right)\cos\left(\omega t - \frac{\omega r}{c}\right)\right.$$
$$\left. - \frac{\cos\theta\sin\left(\omega t - \frac{\omega r}{c}\right)}{r^2} + \frac{\cos\theta}{r\sin\theta}\left(-\sin\theta\right)\sin\left(\omega t - \frac{\omega r}{c}\right) + \frac{1}{r}\left(-\cos\theta\right)\sin\left(\omega t - \frac{\omega r}{c}\right)\right]$$

$$\vec{\nabla}\vec{A} = -\frac{\mu_0 P_0\omega}{4\pi}\cos\theta\left[-\frac{\omega}{rc}\cos\left(\omega t - \frac{\omega r}{c}\right) - \frac{1}{r^2}\sin\left(\omega t - \frac{\omega r}{c}\right)\right]$$

$$\vec{\nabla}\vec{A} = \frac{\mu_0 P_0\omega}{4\pi}\cos\theta\left[\frac{\omega}{rc}\cos\left(\omega t - \frac{\omega r}{c}\right) + \frac{1}{r^2}\sin\left(\omega t - \frac{\omega r}{c}\right)\right]$$

Then $\dfrac{\partial V}{\partial t} = \dfrac{p_0 \cos\theta}{4\pi\varepsilon_0 r}\left[-\dfrac{\omega^2}{c}\cos\left(\omega t - \dfrac{\omega r}{c}\right) - \dfrac{\omega}{r}\sin\left(\omega t - \dfrac{\omega r}{c}\right)\right]$

$$\frac{\partial V}{\partial t} = -\frac{p_0 \cos\theta\, \omega}{4\pi\varepsilon_0}\left[\frac{\omega}{rc}\cos\left(\omega t - \frac{\omega r}{c}\right) + \frac{1}{r^2}\sin\left(\omega t - \frac{\omega r}{c}\right)\right]$$

Comparing the expressions for $\vec{\nabla}\cdot\vec{A}$ and $\frac{\partial V}{\partial t}$ we see that $\vec{\nabla}\cdot\vec{A} = -\mu_0 \varepsilon_0 \frac{\partial V}{\partial t}$ which is the Lorenz condition

GRIFFITHS 11.2

Equation 11.14 $\boxed{V(\vec{r}, t) = -\dfrac{\omega}{4\pi\varepsilon_0 c}\dfrac{\vec{p}_0 \cdot \hat{r}}{r}\sin\left(\omega t - \dfrac{\omega r}{c}\right)}$

Equation 11.17 $\boxed{\vec{A}(r, t) = -\dfrac{\mu_0 \omega}{4\pi r}\vec{p}_0 \sin\left(\omega t - \dfrac{\omega r}{c}\right)}$

Next we note that $\vec{p}_0 \times \hat{r} = p_0 \sin\theta\, \hat{\phi}$

and $\hat{r} \times (\vec{p}_0 \times \hat{r}) = p_0 \sin\theta\, (\hat{r} \times \hat{\phi}) = -p_0 \sin\theta\, \hat{\theta}$

Equation 11.18 $$\boxed{\vec{E}(\vec{r},t) = \frac{\mu_0 \omega^2}{4\pi r}\,\hat{r}\times(\vec{p}_0\times\hat{r})\cos\left(\omega t - \frac{\omega r}{c}\right)}$$

Equation 11.19 $$\boxed{\vec{B}(\vec{r},t) = -\frac{\mu_0 \omega^2}{4\pi c}\,\frac{\vec{p}_0\times\hat{r}}{r}\cos\left(\omega t - \frac{\omega r}{c}\right)}$$

Equation 11.21 $$\boxed{\langle \vec{S}(\vec{r},t)\rangle = \frac{\mu_0 \omega^4}{32\pi^2 c}\,(\vec{p}_0\times\hat{r})^2\,\frac{\hat{r}}{r^2}}$$