PHYSICS 110B HOMEWORK 6 W25

GRIFFITHS 10.3

(a) $\vec{E} = -\vec{\nabla} V - \dfrac{\partial \vec{A}}{\partial t}$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q\,\hat{r}}{r^2}}$$

$\vec{B} = \vec{\nabla} \times \vec{A}$

$$\boxed{\vec{B} = 0}$$

Like a charge at $\vec{r} = 0$ and no $\vec{B}$ field

(b) $V' = V - \dfrac{\partial \lambda}{\partial t} = 0 + \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$

$$\boxed{V' = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$

$$\vec{A}' = \vec{A} + \vec{\nabla}\lambda = -\frac{1}{4\pi\varepsilon_0}\frac{qt\,\hat{r}}{r^2} + \left(-\frac{qt}{4\pi\varepsilon_0}\right)\left(-\frac{\hat{r}}{r^2}\right)$$

$$\boxed{A' = 0}$$

The more obvious potential choice

GRIFFITHS 10.4

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

$$\boxed{\vec{E} = A_0 \omega \cos(kx - \omega t)\, \hat{y}}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\boxed{\vec{B} = A_0 k \cos(kx - \omega t)\, \hat{z}}$$

This is simply a plane wave solutions which we know "works" provided $\boxed{\omega/k = c}$

It is a wave propagating in the $\hat{x}$ direction.

The orientation of the fields in such a wave must be $\vec{B}$ in the direction of $\vec{k} \times \vec{E}$

Since $\hat{x} \times \hat{y} = \hat{z}$, the orientations are also OK

GRIFFITHS 10.6

$\vec{A}$ V : original potentials.

$\vec{A}'$ V' : gauge transformed potentials

$$\vec{A}' = \vec{A} + \vec{\nabla}\lambda \qquad V' = V - \frac{\partial \lambda}{\partial t}$$

Want $\vec{\nabla} A' = -\mu_0 \varepsilon_0 \dfrac{\partial V'}{\partial t}$ – Can I find λ?

$$\vec{\nabla}\left(\vec{A} + \vec{\nabla}\lambda\right) = -\mu_0 \varepsilon_0 \frac{\partial V}{\partial t} + \mu_0 \varepsilon_0 \frac{\partial^2 \lambda}{\partial t^2}$$

$$\underbrace{-\left[\vec{\nabla}\,\vec{A} + \mu_0 \varepsilon_0 \frac{\partial V}{\partial t}\right]}_{\text{call this } f(\vec{r},t)} = \Box^2 \lambda$$

call this $f(\vec{r},t)$

$$\boxed{\Box^2 \lambda = f(\vec{r},t)}$$

This is like 10.16 (a) which we know how to solve

We can always pick $V=0$ by choosing $\lambda(t) = \int V\,dt$

We cannot always pick $\vec{A}=0$ because that would mean $\vec{B} = \text{curl}\,\vec{A} = 0$

GRIFFITHS 10.8

Equation 10.19

$$\frac{d\vec{A}}{dt} = \frac{\partial A}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{A} = (\vec{v} \cdot \vec{\nabla})\vec{A}$$

$$\frac{d\vec{A}}{dt} = -\frac{1}{2}\left[(\vec{v} \cdot \vec{\nabla})(\vec{r} \times \vec{B})\right]$$

$$= -\frac{1}{2}\left[\left(v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z}\right)\left[(yB_z - zB_y)\hat{x} + (zB_x - xB_z)\hat{y} + (xB_y - yB_x)\hat{z}\right]\right]$$

$$= -\frac{1}{2}\left[v_x\left[-B_z\hat{y} + B_y\hat{z}\right] + v_y\left[B_z\hat{x} - B_x\hat{z}\right] + v_z\left[-B_y\hat{x} + B_x\hat{y}\right]\right]$$

$$= -\frac{1}{2}\left[(v_yB_z - v_zB_y)\hat{x} + (-v_xB_z + B_xv_x)\hat{y}\right.$$ .....

continued

$$+\hat{z}\,(v_x B_y - v_y B_x)$$

$$\boxed{\frac{d\vec{A}}{dt} = -\frac{1}{2}(\vec{v}\times\vec{B})}$$

Equation 10.20

$$\frac{d}{dt}(\vec{p}+q\vec{A}) = -q\,\vec{\nabla}(V-\vec{v}\cdot\vec{A})$$

The LHS is

$$\frac{d\vec{p}}{dt} + q\frac{d\vec{A}}{dt} = \frac{d\vec{p}}{dt} - \frac{q}{2}(\vec{v}\times\vec{B})$$

The RHS is

$$-q\,\vec{\nabla}V - \frac{1}{2}q\,\vec{\nabla}\left(\vec{v}\cdot(\vec{r}\times\vec{B})\right) =$$

$$q\vec{E} - \frac{1}{2}q\,\vec{\nabla}\left(\vec{v}\cdot(\vec{r}\times\vec{B})\right)$$

Li wrote $\vec{E} = -q\vec{\nabla}V$ .... magnetostatic & electrostatic

Consider the second term in the RHS

$$\vec{\nabla}\left(\vec{v}\cdot(\vec{r}\times\vec{B})\right) = \vec{\nabla}\left(\vec{r}\cdot(\vec{B}\times\vec{v})\right)$$

using $\vec{a}\cdot(\vec{b}\times\vec{c}) = \vec{b}\cdot(\vec{c}\times\vec{a})$

But for any vector $\vec{\nabla}(\vec{r}\cdot\vec{a}) = \vec{\nabla}\left(\sum r_i a_i\right) = \vec{a}$

This is then $\vec{B}\times\vec{v}$

So RHS is

$$q\vec{E} - \frac{q}{2}(\vec{B}\times\vec{v}) = q\vec{E} + \frac{q}{2}(\vec{v}\times\vec{B})$$

Setting LHS = RHS we have

$$\frac{d\vec{p}}{dt} - \frac{q}{2}(\vec{v}\times\vec{B}) = q\vec{E} + \frac{q}{2}(\vec{v}\times\vec{B})$$

$$\boxed{\frac{d\vec{p}}{dt} = q\left(\vec{E} + (\vec{v}\times\vec{B})\right)}$$