HOMEWORK 5 - W25

GRIFFITHS 9.28

(a) Starting points are

$$\text{curl } \tilde{E} = -\frac{\partial \tilde{B}}{\partial t} = i\omega \tilde{B}_0 \, e^{i(kz-\omega t)}$$

$$\text{curl } \vec{B} = \frac{1}{c^2}\frac{\partial \tilde{E}}{\partial t} = -\frac{i\omega}{c^2} \tilde{E}_0 \, e^{i(kz-\omega t)}$$

Then

$$\boxed{(\text{curl } \tilde{E})_z = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega B_z}$$

$$\boxed{(\text{curl } \tilde{B})_x = \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = \frac{\partial B_z}{\partial y} - ikB_y = -\frac{i\omega}{c^2} E_x}$$

Next take 9.181 as given

Multiply 9.181(iii) by k. This gives

$$ik^2 E_x = i\omega k B_y + \frac{\partial E_z}{\partial x} k \quad Ⓐ$$

Multiply 9.181(v) by ω. This gives

$$-\frac{i\omega^2}{c^2}E_x = -i\omega k B_y + \frac{\partial B_z}{\partial y}\omega \quad Ⓑ$$

Now do Ⓐ+Ⓑ

$$i\left(k^2 - \frac{\omega^2}{c^2}\right)E_x = k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}$$

$$\boxed{E_x = \frac{i}{(\omega/c)^2 - k^2}\left[k\frac{\partial E_z}{\partial x} + \omega\frac{B_z}{\partial y}\right]}$$

Ⓐ $\times \frac{1}{k^2}$ $\quad iE_x = i\frac{\omega}{k}B_y + \frac{1}{k}\frac{\partial E_z}{\partial x}$ Ⓒ

Ⓑ $\times \frac{c^2}{\omega^2}$ $\quad -iE_x = -i\frac{c^2 k}{\omega} + \frac{c^2}{\omega}\frac{\partial B_z}{\partial y}$ Ⓓ

Ⓒ+Ⓓ : $\quad 0 = i\left(\underbrace{\frac{\omega}{k} - \frac{c^2 k}{\omega}}\right)B_y + \frac{1}{k}\frac{\partial E_z}{\partial x} + \frac{c^2}{\omega}\frac{\partial B_z}{\partial y}$

$$\frac{\omega}{k} - \frac{c^2 k}{\omega} = \frac{\omega^2 - c^2k^2}{\omega k} = c^2\left[\frac{(\omega/c)^2 - k^2}{k\omega}\right]$$

$$-i\left[\left(\frac{\omega}{c}\right)^2 - k^2\right]\frac{c^2}{k\omega}B_y = \frac{1}{k}\frac{\partial E_z}{\partial x} + \frac{c^2}{\omega}\frac{\partial B_z}{\partial y}$$

$$-i\left[\left(\frac{w}{c}\right)^2-k^2\right]B_y = \frac{w}{c^2}\frac{\partial E_z}{\partial x}+k\frac{\partial B_z}{\partial y}$$

$$\boxed{B_y = \frac{i}{(w/c)^2-k^2}\left[\frac{w}{c^2}\frac{\partial E_z}{\partial x}+k\frac{\partial B_z}{\partial y}\right]}$$

(b) $\vec{\nabla}\vec{E}=0$

$$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}=0$$

Suppress the $e^{i(kz-wt)}$ from the equations

Note $\frac{\partial E_z}{\partial z}=ikE_z$

Let $A=\frac{i}{(w/c)^2-k^2}$

$$A\left[k\frac{\partial^2E_z}{\partial x^2}+\cancel{w\frac{\partial^2B_z}{\partial x\partial y}}+k\frac{\partial^2E_z}{\partial y^2}-\cancel{w\frac{\partial^2B_z}{\partial x\partial y}}\right]+ikE_z=0$$

$$\frac{i}{(w/c)^2-k^2}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]E_z+iE_z=0$$

$$\boxed{\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\left(\frac{w}{c}\right)^2-k^2\right]E_z=0}$$

Next, let's use $\vec{\nabla}\vec{B}=\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}=0$

$$\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+ikB_z=0$$

$$A\left[k\frac{\partial^2 B_z}{\partial x^2} - \cancel{\frac{\omega}{c^2}\frac{\partial^2 E_z}{\partial x\,\partial y}} + k\frac{\partial^2 B_y}{\partial y^2} + \cancel{\frac{\omega}{c^2}\frac{\partial^2 E_z}{\partial x\,\partial y}}\right] + ikB_z = 0$$

Same algebra as above with $B \longleftrightarrow X$

$$\boxed{\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\omega}{c} - k^2\right]B_z = 0}$$

PROBLEM 9.29

TE_{00} implies that $\boxed{E_z = 0}$ and $k = \frac{\omega}{c}$

From 9.181 (vi) $\frac{\partial B_z}{\partial x} = ikB_x - \frac{i\omega}{c^2}E_y = ik\left[B_x - \frac{E_y}{c}\right]$

From 9.181 (ii)

$$0 = \frac{\partial E_z}{\partial y} = i\omega B_x + ikE_y$$

because $E_z = 0$

$$B_x = -E_y/c$$

So we have $\frac{\partial B_z}{\partial x} = -\frac{2ik}{c}B_y = \frac{2ik}{c^2}E_x$

From 9.181 (v) $\frac{\partial B_z}{\partial y} = ik\left[B_y - \frac{E_x}{c}\right]$

From 9.181 (iii) $ikE_x - \cancel{\frac{\partial E_z}{\partial x}} = i\omega B_y$

$$B_y = \frac{E_x}{c}$$

But since $\frac{\partial B_z}{\partial y} = ik\left[B_y - \frac{E_x}{c}\right] \Rightarrow \frac{\partial B_z}{\partial y} = 0$

Now 9.182 (i) with $\frac{\partial B_z}{\partial y} = 0$ AND $\frac{\partial E_z}{\partial x} = 0$

implies $\boxed{E_x = 0}$

But we had $\frac{\partial B_z}{\partial x} = \frac{2ik}{c^2} E_x = 0$ if $E_x = 0$

Then 9.182 (ii) with $\frac{\partial B_z}{\partial x} = 0$ AND $\frac{\partial E_z}{\partial y} = 0$

Implies $\boxed{E_y = 0}$

Since $E_x = E_y = E_z = 0$, no waves

I guess I did not exactly follow the hint_