PHYSICS 110B FINAL
WINTER 2025

1.

B field of infintely long wire carrying current I

$$B = \frac{\mu_0 I}{2\pi r}$$

where r = distance from wire

Flux $\Phi = \frac{\mu_0 I}{2\pi}\int_a^{a+b} \frac{l\,dr}{r} = \frac{\mu_0 I l}{2\pi}\log\left(\frac{a+b}{b}\right)$

For no induced current, we want Φ to be independent of time

Consider

$$\frac{a+b}{b} = 1 + \frac{a}{b}$$

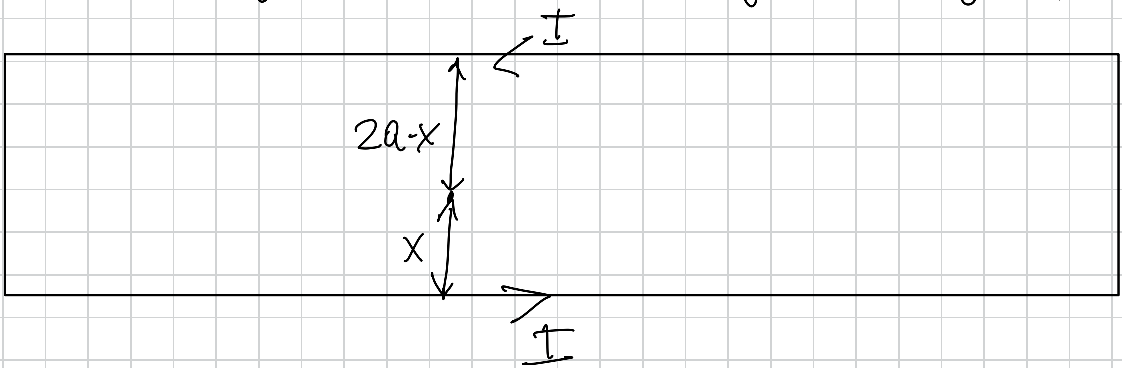
At $t=0$ $\frac{a}{b} = \frac{a_0}{b_0}$

Therefore we want $\frac{a(t)}{b(t)} = \frac{a_0+\alpha t}{b_0+\beta t} = \frac{a_0}{b_0}$

$$a_0+\alpha t = \frac{a_0}{b_0}\left(b_0+\beta t\right) = a_0 + \frac{a_0}{b_0}\beta t$$

This gives $\boxed{\alpha = \frac{a_0}{b_0}\beta}$ or $\boxed{\beta = \frac{b_0\alpha}{a_0}}$

(2) Imagine current flowing in long loop.



The B field at a distance x from the bottom of the loop is

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x} + \frac{1}{2a-x}\right)$$

The small loop is at x between

$x_1 = \frac{1}{2} a$ and $x_2 = \frac{3}{2} a$

The flux through the small loop due to the big loop is

$$\Phi = \frac{\mu_0 I}{2\pi} \int_{x_1}^{x_2} a\, dx \left(\frac{1}{x} + \frac{1}{2a - x} \right)$$

$$\Phi = \frac{\mu_0 I a}{2\pi} \left[\log \frac{x_2}{x_1} - \log \frac{2a - x_2}{2a - x_1} \right]$$

$$\Phi = \frac{\mu_0 I a}{2\pi} \left[\log 3 - \log \frac{1/2\, a}{3/2\, a} \right]$$

$$\Phi = \frac{\mu_0 I a}{\pi} \log 3$$

Since $\Phi = MI$

$$\boxed{M = \frac{\mu_0 a \log 3}{\pi}}$$

③ In S $\quad E_x = \frac{kx\hat{x}}{r^3} \quad E_y = \frac{ky\hat{y}}{r^3} \quad E_z = \frac{kz\hat{z}}{r^3}$

where $k = \frac{q}{4\pi\varepsilon_0} \quad r^2 = x^2 + y^2 + z^2$

In S' $\quad E'_x = E_x \quad E'_y = \gamma E_y \quad E_z = \gamma E_z$

Also $y' = y$ and $z' = z$

But $x' = \gamma(x - vt)$

or $x = \gamma(x' + vt')$

So
$$r^2 = \gamma^2(x' + vt')^2 + y'^2 + z'^2$$

$$\boxed{\begin{aligned} E'_x &= \frac{q}{4\pi\varepsilon_0} \frac{\gamma(x'+vt')\hat{x}}{(\gamma^2(x'+vt')^2 + y'^2 + z'^2)^{3/2}} \\ E'_y &= \frac{q}{4\pi\varepsilon_0} \frac{y'\gamma\hat{y}}{(\gamma^2(x'+vt')^2 + y'^2 + z'^2)^{3/2}} \\ E'_z &= \frac{q}{4\pi\varepsilon_0} \frac{z'\gamma\hat{z}}{(\gamma^2(x'+vt')^2 + y'^2 + z'^2)^{3/2}} \end{aligned}}$$

4 Conservation of energy $(c=1)$

$$E_H + E_Z = E$$

Conservation of momentum $\vec{P}_H = -\vec{P}_Z$

Write $|\vec{P}_H| = |\vec{P}_Z| = p$

Since $E_H = \sqrt{p^2 + M_H^2}$ and $E_Z = \sqrt{p^2 + M_Z^2}$ we have

$$\sqrt{p^2 + M_H^2} + \sqrt{p^2 + M_Z^2} = E$$

$$\sqrt{p^2 + M_H^2} = E - \sqrt{p^2 + M_Z^2}$$

$$\cancel{p^2} + M_H^2 = E^2 + \cancel{p^2} + M_Z^2 - 2E\sqrt{p^2 + M_Z^2}$$

$$2E\sqrt{p^2 + M_Z^2} = E^2 + M_Z^2 - M_H^2$$

$$p^2 + M_Z^2 = \frac{(E^2 + M_Z^2 - M_H^2)^2}{4E^2}$$

$$p^2 = \frac{(E^2 + M_Z^2 - M_H^2)^2}{4E^2} - M_Z^2$$

$$p^2 = \frac{E^4 + M_Z^4 + M_H^4 - 2M_Z^2M_H^2 + 2E^2(M_Z^2 - M_H^2) - 4E^2M_Z^2}{4E^2}$$

$$p^2 = \frac{E^4 + M_Z^4 + M_H^4 - 2M_Z^2M_H^2 - 2E^2(M_Z^2 + M_H^2)}{4E^2}$$

$$\boxed{p = \frac{\sqrt{E^4 + M_Z^4 + M_H^4 + 2E^2(M_Z^2 + M_H^2) - 2M_Z^2M_H^2}}{2E}}$$

5) I = current through the object

J = current density $= \dfrac{I}{\pi(b^2 - a^2)}$

Ohms Law

$$J = \sigma E = E/\rho = \frac{1}{\rho}\frac{V}{\ell}$$

$$\frac{I}{\pi(b^2 - a^2)} = \frac{1}{\rho}\frac{V}{\ell}$$

$$\frac{V}{I} = \frac{\rho \ell}{\pi(b^2 - a^2)}$$

But Ohms law $R = V/I \Rightarrow$ $\boxed{R = \dfrac{\rho \ell}{\pi(b^2 - a^2)}}$

(6) (a) We use $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Let's take the curl

$$\vec{\nabla} \times \vec{E} = \frac{E_0 \pi e^{i\omega t}}{a} \left[\sin\frac{\pi y}{a} \cos\frac{\pi z}{a} \hat{y} - \cos\frac{\pi y}{a} \sin\frac{\pi z}{a} \hat{z} \right]$$

Since $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

we have

$$\boxed{\begin{aligned} B_y &= -\frac{E_0 \pi e^{i\omega t}}{i a \omega} \sin\frac{\pi y}{a} \cos\frac{\pi z}{a} \\ B_z &= +\frac{E_0 \pi e^{i\omega t}}{i a \omega} \cos\frac{\pi y}{a} \sin\frac{\pi z}{a} \\ B_x &= 0 \end{aligned}}$$

(b) Now we are going to use the fact that $\vec{\nabla} \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$

$$(\vec{\nabla}\times\vec{B})_x = \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}$$

$$(\vec{\nabla}\times\vec{B})_x = -\frac{2E_0\pi^2}{iQ^2\omega} e^{i\omega t}\left[\sin\frac{\pi y}{a}\sin\frac{\pi z}{a}\right]$$

$$\frac{1}{c^2}\frac{\partial E_x}{\partial t} = \frac{i\omega E_0}{c^2} e^{i\omega t}\left(\sin\frac{\pi y}{Q}\sin\frac{\pi z}{a}\right)$$

We must then have

$$\frac{i\omega E_0}{c^2} = -\frac{2E_0\pi^2}{iQ^2\omega}$$

which gives

$$\boxed{\omega = \sqrt{2}\pi c/a}$$

$\boxed{7: A}$ $\boxed{8: A}$ $\boxed{9: B}$