Winter 2025, Physics 110B Final

Remember to write your name and your perm number in the front of your blue book.

At the end of the exam only turn in the blue book. Keep all other papers (your cheat sheets, your scratch notes, and the exam questions) for yourself.

Griffiths front cover

VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient: $\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$

Divergence: $\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

Curl: $\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$

Laplacian: $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient: $\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$

Divergence: $\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$

Curl:
$$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian: $\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient: $\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$

Divergence: $\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$

Curl: $\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$

Laplacian: $\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

(2) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$

(5) $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$

(6) $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

(7) $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$

(8) $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$

Second Derivatives

(9) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

(10) $\nabla \times (\nabla f) = 0$

(11) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem: $\int_{\mathbf{a}}^{\mathbf{b}} (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem: $\int (\nabla \cdot \mathbf{A})\, d\tau = \oint \mathbf{A} \cdot d\mathbf{a}$

Curl Theorem: $\int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$

Griffiths back cover

BASIC EQUATIONS OF ELECTRODYNAMICS

Maxwell's Equations

In general:

$$\begin{cases} \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{cases}$$

In matter:

$$\begin{cases} \nabla \cdot \mathbf{D} = \rho_f \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \end{cases}$$

Auxiliary Fields

Definitions:

$$\begin{cases} \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \end{cases}$$

Linear media:

$$\begin{cases} \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}, \quad \mathbf{D} = \epsilon \mathbf{E} \\ \mathbf{M} = \chi_m \mathbf{H}, \quad \mathbf{H} = \frac{1}{\mu}\mathbf{B} \end{cases}$$

Potentials

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

Lorentz force law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Energy, Momentum, and Power

Energy: $U = \frac{1}{2}\int \left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) d\tau$

Momentum: $\mathbf{P} = \epsilon_0 \int (\mathbf{E} \times \mathbf{B})\, d\tau$

Poynting vector: $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$

Larmor formula: $P = \frac{\mu_0}{6\pi c} q^2 a^2$

FUNDAMENTAL CONSTANTS

ϵ_0	$= 8.85 \times 10^{-12}\ \mathrm{C^2/Nm^2}$	(permittivity of free space)
μ_0	$= 4\pi \times 10^{-7}\ \mathrm{N/A^2}$	(permeability of free space)
c	$= 3.00 \times 10^{8}\ \mathrm{m/s}$	(speed of light)
e	$= 1.60 \times 10^{-19}\ \mathrm{C}$	(charge of the electron)
m	$= 9.11 \times 10^{-31}\ \mathrm{kg}$	(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{r}} + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{r}} + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}} \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \tan^{-1}\left(\sqrt{x^2+y^2}/z\right) \\ \phi = \tan^{-1}(y/x) \end{cases} \qquad \begin{cases} \hat{\mathbf{r}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \end{cases}$$

Cylindrical

$$\begin{cases} x = s\cos\phi \\ y = s\sin\phi \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{x}} = \cos\phi\,\hat{\mathbf{s}} - \sin\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} = \sin\phi\,\hat{\mathbf{s}} + \cos\phi\,\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\begin{cases} s = \sqrt{x^2+y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases} \qquad \begin{cases} \hat{\mathbf{s}} = \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}} \\ \hat{\boldsymbol{\phi}} = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

Maxwell Equations in integral form:

$$\int_S \mathbf{D}\cdot d\mathbf{a} = Q_{fenclosed} \qquad \int_S \mathbf{B}\cdot d\mathbf{a} = 0$$

$$\int_P \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{a} \qquad \int_P \mathbf{H}\cdot d\mathbf{l} = I_{enclosed} + \frac{d}{dt}\int_S \mathbf{D}\cdot d\mathbf{a}$$

Boundary conditions at the interface of two materials.

$$D_1^{\perp} - D_2^{\perp} = \sigma_f \qquad B_1^{\perp} - B_2^{\perp} = 0$$

$$\mathbf{E_1^{\|}} - \mathbf{E_2^{\|}} = 0 \qquad \mathbf{H_1^{\|}} - \mathbf{H_2^{\|}} = \mathbf{K}_f \times \hat{\mathbf{n}}$$

Wave equation in vacuum:

$$\nabla^2\mathbf{E} = \mu_0\epsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2} \qquad \nabla^2\mathbf{B} = \mu_0\epsilon_0\frac{\partial^2\mathbf{B}}{\partial t^2}$$

Speed of light in vacuum $c = 1/\sqrt{\epsilon_0\mu_0}$. In dielectric $v = \frac{c}{n}$ where $n = \sqrt{\epsilon\mu/\epsilon_0\mu_0}$.

Plane wave solution propagating in direction $\hat{\mathbf{k}}$ (not in metals)

$$\mathbf{E} = \mathbf{E_0}\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta) \qquad \mathbf{B} = \frac{1}{v}\hat{\mathbf{k}}\times\mathbf{E}$$

with $\mathbf{E_0}$ perpendicular to $\hat{\mathbf{k}}$ and $v = \omega/k$.

Snell's law: $\frac{\sin\theta_T}{\sin\theta_I} = \frac{n_1}{n_2}$.

For linearly polarized waves with polarization in the material boundary plane:

$$E_{0R} = \frac{\alpha-\beta}{\alpha+\beta}E_{0I} \qquad E_{0T} = \frac{2}{\alpha+\beta}E_{0I} \qquad \alpha = \frac{\cos\theta_T}{\cos\theta_I} \qquad \beta = \frac{\mu_1 v_1}{\mu_2 v_2}$$

Wave equation in metal:

$$\nabla^2\mathbf{E} = \mu\epsilon\frac{\partial^2\mathbf{E}}{\partial t^2} + \mu\sigma\frac{\partial\mathbf{E}}{\partial t} \qquad \nabla^2\mathbf{B} = \mu_0\epsilon_0\frac{\partial^2\mathbf{B}}{\partial t^2} + \mu\sigma\frac{\partial\mathbf{B}}{\partial t}$$

Solution for propagation in z direction:

$$\tilde{\mathbf{E}} = \tilde{\mathbf{E}}_0\ e^{-\kappa z}e^{i(kz-\omega t)} \qquad \tilde{\mathbf{B}} = \tilde{\mathbf{B}}_0\ e^{-\kappa z}e^{i(kz-\omega t)}$$

$$k = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}+1\right]^{1/2} \qquad \kappa = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}-1\right]^{1/2}$$

Maxwell equations written in terms of potentials:

$$\nabla^2 V + \frac{\partial}{\partial t}(\nabla\cdot\mathbf{A}) = -\frac{\rho}{\epsilon_0}$$

$$(\nabla^2\mathbf{A} - \mu_0\epsilon_0\frac{\partial^2\mathbf{A}}{\partial t^2}) - \nabla(\nabla\cdot\mathbf{A} + \mu_0\epsilon_0\frac{\partial V}{\partial t}) = -\mu_0\mathbf{J}$$

Gauge transformations: $\mathbf{A}' = \mathbf{A} + \nabla\lambda$ and $V' = V - \frac{\partial\lambda}{\partial t}$.

Lorenz gauge: $\nabla\cdot\mathbf{A} = -\mu_0\epsilon_0\frac{\partial V}{\partial t}$ Coulomb gauge $\nabla\cdot\mathbf{A} = 0$.

In Lorenz gauge: $\Box^2 V = -\rho/\epsilon_0$ $\Box^2\mathbf{A} = -\mu_0\mathbf{J}$

Retarded potentials:

$$V(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}',t_r)}{\mathscr{r}} d\tau' \qquad\qquad \mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}',t_r)}{\mathscr{r}} d\tau'$$

Electric and magnetic fields (Jefimenko equations):

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\int \left[\frac{\rho(\mathbf{r}',t_r)}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}} + \frac{\dot{\rho}(\mathbf{r}',t_r)}{c\mathscr{r}}\hat{\boldsymbol{\mathscr{r}}} + \frac{\dot{\mathbf{J}}(\mathbf{r}',t_r)}{c^2\mathscr{r}}\right] d\tau'$$

$$\mathbf{B}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int \left[\frac{\mathbf{J}(\mathbf{r}',t_r)}{\mathscr{r}^2} + \frac{\dot{\mathbf{J}}(\mathbf{r}',t_r)}{c\mathscr{r}}\right] \times \hat{\boldsymbol{\mathscr{r}}}\, d\tau'$$

Lienard-Wiechter potentials:

$$V(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\frac{qc}{c\mathscr{r} - \boldsymbol{\mathscr{r}}\cdot\mathbf{v}} \qquad\qquad \mathbf{A}(\mathbf{r},t) = \frac{V(\mathbf{r},t)}{c^2}\mathbf{v}$$

Field of a moving point charge ($\mathbf{u} \equiv c\hat{\boldsymbol{\mathscr{r}}} - \mathbf{v}$):

$$\mathbf{E}(\mathbf{r},t) = \frac{q}{4\pi\epsilon_0}\frac{\mathscr{r}}{(\boldsymbol{\mathscr{r}}\cdot\mathbf{u})^3}\left[(c^2-v^2)\mathbf{u} - \boldsymbol{\mathscr{r}}\times(\mathbf{u}\times\mathbf{a})\right] \qquad\qquad \mathbf{B}(\mathbf{r},t) = \frac{1}{c}\hat{\boldsymbol{\mathscr{r}}}\times\mathbf{E}(\mathbf{r},t)$$

Electric dipole radiation, approximate formula. (Note: $\ddot{\mathbf{p}}$ at retarded time):

$$\mathbf{E}(\mathbf{r},t) = \frac{\mu_0}{4\pi r}\left[\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\ddot{\mathbf{p}})\right] \qquad\qquad \mathbf{B}(\mathbf{r},t) = -\frac{\mu_0}{4\pi rc}(\hat{\mathbf{r}}\times\ddot{\mathbf{p}})$$

Larmor formula: $P = \frac{\mu_o q^2 a^2}{6\pi c}$

Lienard formula: $P = \frac{\mu_0 q^2\gamma^6}{6\pi c}\left(a^2 - \frac{1}{c^2}|\mathbf{v}\times\mathbf{a}|^2\right)$

Boost with velocity v in x direction:

$$\begin{aligned} x' &= \gamma(x - vt)\\ y' &= y\\ z' &= z\\ t' &= \gamma(t - \beta x/c)\end{aligned}$$

Time-position and energy-momentum contravariant four vectors:

$$\begin{aligned} x^\mu &: (ct;\mathbf{r})\\ p^\mu &: (E/c;\mathbf{p})\end{aligned}$$

Contravariant (a^μ)to covariant (a_μ) relations: $a_0 = -a^0$ and $a_i = a^i$ for $i = 1,2,3$.
Lorentz invariant: $a_\mu b_\mu$. In particular $p_\mu p^\mu = -E^2/c^2 + p^2$ leads to $E^2 = m^2c^4 + p^2c^2$.
Other important relationships: $E = \gamma mc^2$ and $\mathbf{p} = \gamma m\mathbf{v}$.

Lorentz transformation matrix (boost velocity in x direction);

$$\Lambda^\mu_\nu = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0\\ -\gamma\beta & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Transformation law for forur vectors: $a^\mu \to \Lambda^\mu_\nu a^\nu$

Transformation law for EM fields:

$$\mathbf{E}_\parallel \to \mathbf{E}_\parallel$$
$$\mathbf{B}_\parallel \to \mathbf{B}_\parallel$$
$$\mathbf{E}_\perp \to \gamma(\mathbf{E}_\perp + \mathbf{v} \times \mathbf{B}_\perp)$$
$$\mathbf{B}_\perp \to \gamma(\mathbf{B}_\perp - \frac{\mathbf{v}}{c^2} \times \mathbf{E}_\perp)$$

EM Field tensor

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{bmatrix}$$

The (dual) $G^{\mu\nu}$ tensor is obtained from the $F^{\mu\nu}$ tensor by swapping $\mathbf{E}/c$ with $\mathbf{B}$ and $\mathbf{B}$ with $-\mathbf{E}/c$.

The transformation law is $F^{\mu\nu} \to \Lambda^\mu_\alpha \Lambda^\nu_\beta F^{\alpha\beta}$ and similarly for $G^{\mu\nu}$.

Covariant and contravariant partial derivatives: $\partial_\mu = \frac{\partial}{\partial x^\mu}$ and $\partial^\mu = \frac{\partial}{\partial x_\mu}$.
Maxwell equations with $J^\mu = (c\rho; \mathbf{J})$: $\partial_\nu F^{\mu\nu} = \mu_0 J^\mu$ and $\partial_\nu G^{\mu\nu} = 0$.

Continuity equation: $\partial_\mu J^\mu = 0$

Four potential: $A^\mu = (V/c; \mathbf{A})$.

Field tensor from potential: $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$.

Winter Quarter 2025
UCSB Physics 110b Final

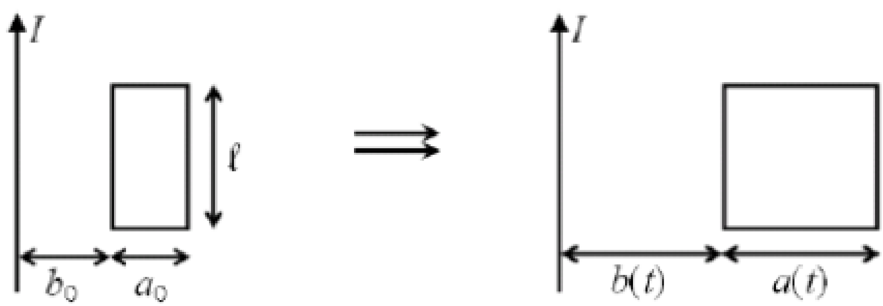
Problems 1 through 6 are worth 10 points each.
The multiple choice GRE questions 7,8 and 9 are worth 3.333... points each.
For the multiple choice questions, just give the answer (a, a, x....) no need to explain. For the GRE questions, points are awarded for correct answers and not deducted for wrong answers.

- **Problem 1**

  A rectangular loop of width a_0 and length ℓ sits a distance b_0 away from an infinite straight wire carrying a constant current I. The wire and the loop lie in the same plane. At $t = 0$ two things happen simultaneously (see picture):

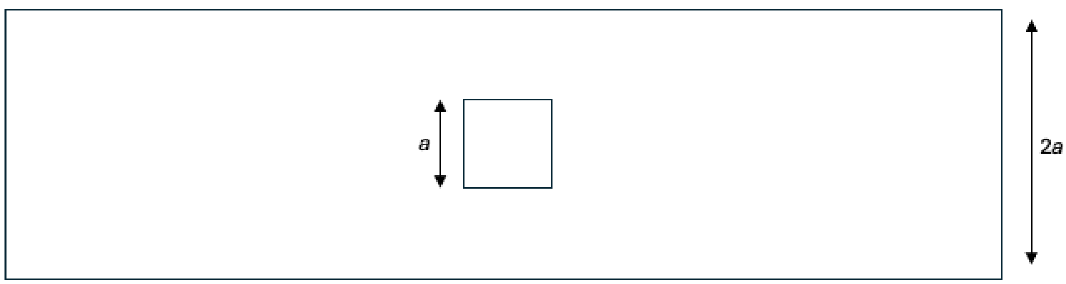
  1. The loop is moved directly away from the wire at a constant rate β, *i.e.*, $b(t) = b_0 + \beta t$.
  2. The loop is stretched at a constant rate α, *i.e.*, $a(t) = a_0 + \alpha t$. The length ℓ is unchanged.

  Find the relationship between α and β such that there will be no induced current in the loop.



- **Problem 2**

  A very very long (assume infinitely long) rectangular loop of wire has width $2a$. At the center of this loop there is a square loop of wire of side-length a. What is the mutual inductance between the two loops? (The two loops lie in the same plane).



- **Problem 3**

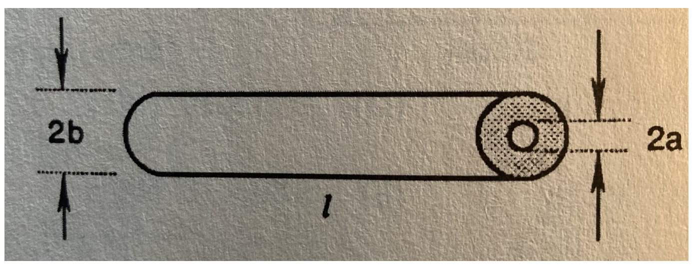
  A point charge q is at rest at the origin in a reference system S. A reference system S' moves to the right (*i.e.*, to positive x) at speed v_0 with respect to S. What is the electric field in S'. Express your answer in Cartesian coordinates $\mathbf{E}' = E'_x\hat{\mathbf{x}} + E'_y\hat{\mathbf{y}} + E'_z\hat{\mathbf{z}}$, and give the components of $\mathbf{E}'$ as a function of the primed coordinates t', x', y', and z'.
  PS: the "origins" of the two systems overlap, *i.e.*, at $t = t' = 0$ we have $x = x' = 0$, $y = y' = 0$, and $z = z' = 0$.

- **Problem 4**

  CERN is hoping to build a new accelerator (FCC-ee) to collide electrons and positrons head-on, *i.e.*, with equal and opposite momentum, with a total energy (electron + positron) as high as 350 GeV. One of the goals is to study the properties of Higgs bosons (H) using the the process $e^+e^- \to HZ$. For a total collision energy E, what is the momentum of the Higgs boson given its mass M_H and the mass of the Z boson M_Z?

- **Problem 5**

  A resistor is made from a hollow cyclinder of length l, inner radius a and outer radius b. The region $a < r < b$ is filled with a material of resistivity ρ. Find the resistence of this object.



- **Problem 6**

  The electric field in a metal box with dimensions a, a, and $2a$ in the x, y, and z directions has components

$$E_x = E_0 \sin(\pi y/a)\sin(\pi z/a)e^{i\omega t}$$
$$E_y = E_z = 0$$

  (a) Find the magnetic field inside the box (you can leave your answer in complex notation).
  (b) What is the value of ω?

- **Problem 7**

  A plane wave solution of Maxwell's equation in free space is

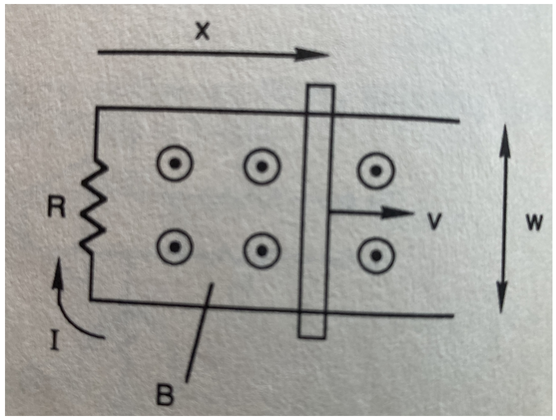
$$\mathbf{E} = \hat{\mathbf{y}}E_{0y}\cos(\omega t - kx + \alpha) + \hat{\mathbf{z}}E_{0z}\cos(\omega t - kx + \beta)$$

  Let $\delta = \beta - \alpha$. Under what conditions do we have elliptic polarization.

  (a) $\delta = \pm\pi/2$
  (b) $\delta = 0$
  (c) $\delta = \pm\pi/4$ and $E_{0z} = E_{0y}$
  (d) $\delta = \pm\pi/2$ and $E_{0z} = E_{0y}$
  (e) $\delta = \pm\pi$

- **Problem 8**

  Given that $B = 1$ T, $R = 2\ \Omega$, and $w = 0.5$ m, what is the speed of the sliding bar if the current is $I = 0.5$?



  (a) 2 m/s
  (b) 4 m/s
  (c) 1 m/s
  (d) 3 m/s
  (e) 5 m/s

- **Problem 9**

  Which of the following Maxwell's equations implies that there are no magnetic monopoles.

  (a) $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$
  (b) $\nabla \cdot \mathbf{B} = 0$
  (c) $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
  (d) $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$
  (e) Magnetic monopoles have recently been discovered