SESSION 9

WORK IN NATURAL UNITS

By conservation of momentum particle 1 and particle 2 have equal and opposite momentum (magnitude = p)

Initial 4 vector = $(M, \vec{0})$ =

Final 4 vector = $(E_1 + E_2, \vec{0})$

$E_1 = \sqrt{p^2 + m_1^2}$ $E_2 = \sqrt{p^2 + m_2^2}$

Conservation of 4 momentum (or conservation of energy)

$$\sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} = M$$

$$\sqrt{p^2 + m_1^2} = M - \sqrt{p^2 + m_2^2}$$

$$p^2 + m_1^2 = M^2 + p^2 + m_2^2 - 2M\sqrt{p^2 + m_2^2}$$

$$2M\sqrt{p^2 + m_2^2} = M^2 + m_2^2 - m_1^2$$

$$4M^2\left(p^2+m_2^2\right) = \left(M^2+m_2^2-m_1^2\right)^2$$

$$p^2 = \frac{\left(M^2+m_2^2-m_1^2\right)^2}{4M^2} - m_2^2$$

Then since $E_2 = \sqrt{p^2+m_2^2}$

$$\boxed{E_2 = \frac{M^2+m_2^2-m_1^2}{2M}}$$

For E_1, I can just interchange $1 \leftrightarrow 2$

$$\boxed{E_1 = \frac{M^2+m_1^2-m_2^2}{2M}}$$

As a sanity check, I should find $E_1+E_2=M$

$$E_1+E_2 = \frac{M^2+m_1^2-m_2^2}{2M} + \frac{M^2+m_2^2-m_1^2}{2M}$$

$$E_1+E_2 = \frac{2M^2}{2M} = M \quad \checkmark \text{ OK!}$$