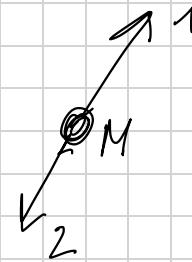


SESSION 9



WORK IN
NATURAL
UNITS

By conservation of momentum

particle 1 and particle 2 have equal and opposite momentum (magnitude = P)

Initial 4 vector = $(M, \vec{0})$ =

Final 4 vector = $(E_1 + E_2, \vec{0})$

$$E_1 = \sqrt{P^2 + M_1^2} \quad E_2 = \sqrt{P^2 + M_2^2}$$

Conservation of 4 momentum (or conservation of energy)

$$\sqrt{P^2 + M_1^2} + \sqrt{P^2 + M_2^2} = M$$

$$\sqrt{P^2 + M_1^2} = M - \sqrt{P^2 + M_2^2}$$

$$P^2 + M_1^2 = M^2 + P^2 + M_2^2 - 2M\sqrt{P^2 + M_2^2}$$

$$2M\sqrt{P^2 + M_2^2} = M^2 + M_2^2 - M_1^2$$

$$4M^2(P^2 + M_2^2) = (M^2 + M_2^2 - M_1^2)^2$$

$$P^2 = \frac{(M^2 + M_2^2 - M_1^2)^2}{4M^2} - M_2^2$$

Then since $E_2 = \sqrt{P^2 + M_2^2}$

$$E_2 = \frac{M^2 + M_2^2 - M_1^2}{2M}$$

For E_1 , I can just interchange $1 \leftrightarrow 2$

$$E_1 = \frac{M^2 + M_1^2 - M_2^2}{2M}$$

As a sanity check, I should find $E_1 + E_2 = M$

$$E_1 + E_2 = \frac{M^2 + M_1^2 - M_2^2}{2M} + \frac{M^2 + M_2^2 - M_1^2}{2M}$$

$$E_1 + E_2 = \frac{2M^2}{2M} = M$$

OK!