Physics 110B, Discussion Session 5

The optical length of the path traversed by a light ray traveling bewteen the points P and Q is defined by the line integral along the path

$$L = \int_P^Q n \, d\ell$$

where n is the index of refraction along the path.

Fermat's principle states that the actual path length is the one such that L is an extreme (usually a minimum). If n is constant, the path that minimizes L is a straight line. Note that minimizing the optical path is the same as minimizing the time taken to go from P to Q since $n \, d\ell = c/v \, d\ell = c/(d\ell/dt) \, d\ell = c \, dt$.

Let the xy plane be the boundary between two dielectrics, index of refraction n_1 for $z < 0$, n_2 for $z > 0$. Let P and Q have coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) with $z_1 < 0$. and $z_2 > 0$. The (PQ) path will cross the xy plane at some point $(x, y, z = 0)$.

Show that according to Fermat's principle

1. The path will lie in a plane (the plane of incidence)

2. Snell's law is satisfied.

Hint: with no loss of generality you can choose the axes such that $y_1 = y_2 = 0$.