SESSION 3

GRIFFITHS 8.28

(a) each q has $\vec{v} = \omega R\hat{\phi} + v_z\hat{z}$

$$\vec{F}_i = q(\vec{v}\times\vec{B}) = qk(2\omega Rz\hat{\rho} - Rv_z\hat{\phi} + \omega R^2\hat{z})$$

on each charge

Net force is $\vec{F} = \sum_{i=1}^{n}\vec{F}_i$

As we go around the circle, the $\hat{\rho}$ and $\hat{\phi}$ forces cancel out, and all we are left with is the $\hat{z}$ component

$$nqk\omega R^2\hat{z} = M\frac{d^2z}{dt^2}\hat{z}$$

So we have

$$\frac{d^2z}{dt^2} = \frac{nqkR^2}{m}\omega(t)$$

In order to solve this we need $\omega(t)$.

Consider now the torque

$$\vec{N} = \sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i = nR\hat{\rho} \times (-qkRv_z)\hat{\phi}$$

$$\vec{N} = -nqkR^2 v_z = I \frac{d\omega}{dt} \hat{z}$$ I = moment of inertia

This then gives
$$\frac{d\omega}{dt} = -\frac{nqkR^2}{I} \frac{dz}{dt}$$

$$\frac{d^2\omega}{dt^2} = -\frac{nqkR^2}{I} \frac{d^2z}{dt}$$

Then using the equation circled in black:

$$\frac{d^2\omega}{dt^2} = -\frac{(nqkR^2)^2}{MI} \omega$$

and since $I = \frac{1}{2} MR^2$

$$\frac{d^2\omega}{dt^2} = -2\,\frac{n^2q^2k^2R^2}{M^2}\,\omega$$

Let $\beta^2 = 2\dfrac{n^2q^2k^2R^2}{M^2}$ $\Rightarrow$ $\boxed{\omega(t) = \omega_0 \cos\beta t}$

Going back to the equation circled in red

$$\frac{dz}{dt} = -\frac{I}{nqkR^2}\frac{d\omega}{dt} = -\frac{\frac{1}{2}mR^2}{nqkR^2}\,\beta\,\omega_0\,(-\sin\beta t)$$

$nqk = \dfrac{\beta R}{\sqrt{2}}$

$$\frac{dz}{dt} = \frac{1}{\sqrt{2}}\frac{R}{M}\,\omega_0 \sin\beta t$$

$$\boxed{z(t) = \frac{R\,\omega_0}{\sqrt{2}\,M}\,(1-\cos\beta t) \quad \text{if } z(0)=0}$$

Check conservation of energy

$$\frac{1}{2}\,m\,v_z(t) + \frac{1}{2}\,I\,\omega^2(t) = E$$

But $v_z(t) = \dfrac{dz}{dt} = \dfrac{1}{\sqrt{2}}\dfrac{R}{m}\,\omega_0 \sin\beta t$

$$E = \frac{1}{2} m \frac{R^2}{2M^2} \omega_0^2 \sin^2 \beta t + \frac{1}{2} I \omega_0^2 \cos^2 \beta t$$

$$E = \frac{1}{2} \frac{MR^2}{2} \omega_0 \sin^2 \beta t + \frac{1}{2} \left(\frac{MR^2}{2}\right) \omega_0^2 \cos^2 \beta t$$

$$\boxed{E = \frac{1}{2} \frac{MR^2}{2} \omega_0^2 = \frac{1}{2} I \omega_0^2}$$