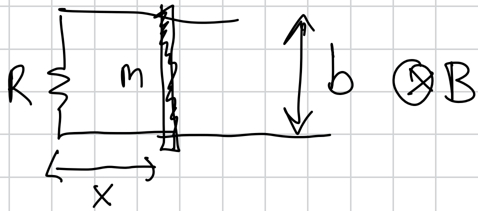
PHYSICS 110B - WINTER 2025
MIDTERM SOLUTIONS

PROBLEM 1



$$\Phi = B\,x(t)\,b \qquad \frac{d\Phi}{dt} = Bb\frac{dx}{dt} = Bb\,v(t) = -\text{emf}$$

$$I(t) = \frac{\text{emf}}{R} = -\frac{Bb\,v(t)}{R}$$

If B is into the paper, the direction of the current is clockwise, ie upwards into the bar

There is a force on the moving bar which is in magnitude $F = IbB = b^2B^2v/R$

The direction of this force is to the left, i.e. it opposes the motion of the bar to the right

$$F = -m\frac{dv}{dt}$$

$$\frac{b^2B^2}{R}v = -m\frac{dv}{dt}$$

Define $\boxed{T = \frac{mR}{B^2 b^2}}$ $\frac{v}{T} = -\frac{dv}{dt}$

The solution, that insures $v(t=0) = v_0$ is

$$\boxed{v = v_0 e^{-t/T}}$$

PROBLEM 2

$$\vec{E} = E_0 (\hat{x} + \hat{y}) \cos \frac{2\pi}{\lambda}(z + ct)$$

This is a plane wave propagating in the negative $\hat{z}$ direction with speed c

$\vec{B} = \frac{1}{c} \hat{k} \times \vec{E}$ where $\hat{k} = -\hat{z}$

$$-\hat{z} \times (\hat{x} + \hat{y}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix} = \hat{x} - \hat{y}$$

$$\boxed{\vec{B} = \frac{E_0}{c} (\hat{x} - \hat{y}) \cos \frac{2\pi}{\lambda}(z + ct)}$$

PROBLEM 3

From the cheat-sheet

$$K = \omega \sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2} - 1\right]^{1/2}$$

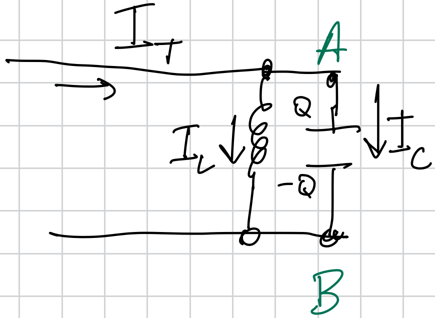
$\sigma \gg \varepsilon\omega$ leads to

$$K \simeq \omega \sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{\left(\frac{\sigma}{\varepsilon\omega}\right)^2} - 1\right]^{1/2} = \omega \sqrt{\frac{\varepsilon\mu}{2}}\left[\frac{\sigma}{\varepsilon\omega} - 1\right]^{1/2}$$

$$K \simeq \omega \sqrt{\frac{\varepsilon\mu}{2}} \sqrt{\frac{\sigma}{\varepsilon\omega}} = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\boxed{\text{Skin depth} = \frac{1}{K} = \sqrt{\frac{2}{\omega\mu\sigma}}}$$

PROBLEM 4



$$I_c = \frac{dQ}{dt}$$

$$V_A - V_B = \frac{Q}{C} \qquad V_A - V_B = L\frac{dI_L}{dt}$$

$$\frac{Q}{C} = L\frac{dI_L}{dt}$$

$$\frac{1}{C}\frac{dQ}{dt} = L\frac{d^2 I_L}{dt^2}$$

$$I_C = LC\frac{d^2 I_L}{dt^2}$$

Let $\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$

Also $I_C = I_T - I_L$

$$\boxed{I_T = \frac{1}{\omega_0^2}\frac{d^2 I_L}{dt^2} + I_L} \quad (1)$$

Let's work with complex numbers.

Try solution $I_L = \tilde{I}_{0L} e^{i\omega t}$

$$I_0 e^{i\omega t} = -\frac{\omega^2}{\omega_0^2}\tilde{I}_{0L} e^{i\omega t} + \tilde{I}_{0L} e^{i\omega t}$$

Gives $\tilde{I}_0 = \frac{\omega_0^2 - \omega^2}{\omega_0^2}\tilde{I}_{0L}$

$$\tilde{I}_{0L} = \frac{\omega^2}{\omega_0^2 - \omega^2} \tilde{I}_0$$

$$\boxed{I_L = \frac{\omega_0^2}{\omega_0^2 - \omega^2} I_0 e^{i\omega t}}$$

$$\boxed{\omega_0^2 = 1/LC}$$

And if I wanted to take the real part

$$\boxed{I_L = \frac{\omega_0^2}{\omega_0^2 - \omega^2} I_0 \cos\omega t}$$

I can equally well solve equation (1) without using complex numbers

Assume

$$I_L = A\cos\omega t + B\sin\omega t$$

$$\frac{d^2 I_L}{dt^2} = -\omega^2(A\cos\omega t + B\sin\omega t)$$

Then equation (1) becomes

$$I_0\cos\omega t = -\frac{\omega^2}{\omega_0^2}(A\cos\omega t + B\sin\omega t) + (A\cos\omega t + B\sin\omega t)$$

For this to work at all times t we need $\boxed{B=0}$

Then, the $\cos\omega t$ cancels, and we get

$$I_0 = -\frac{\omega^2}{\omega_0^2} A + A = \left(\frac{\omega_0^2 - \omega^2}{\omega_0^2}\right) A$$

$$\boxed{A = \frac{\omega_0^2}{\omega_0^2 - \omega^2}}$$

$$\boxed{I_L = \frac{\omega_0^2}{\omega_0^2 - \omega^2} \cos\omega t}$$