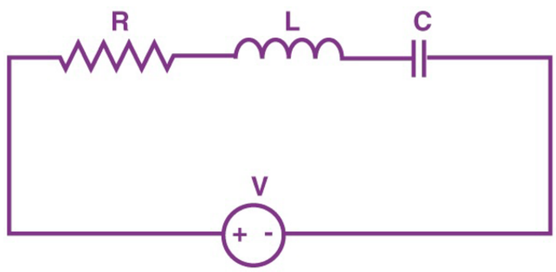
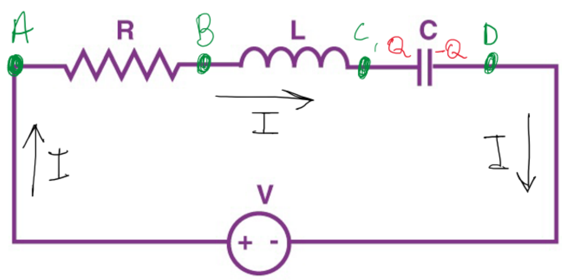
How to use Kirchoff's voltage law in a circuit

Claudio C.

I have fielded a few questions on how to get all the signs right when applying Kirchoff's law to a circuit that includes batteries, capacitors, resistors, and inductors. It is indeed easy to get the signs wrong. My advice is to be **very** pedantic, label everything in sight, and do not skip steps. For example, consider the circuit below.



Here is how I would label it



Let's review what I did

- I labelled all nodes in the circuit (A, B, C, D).
- I picked a sign convention for the current I: positive current is defined as flowing in the clockwise direction. This choice is entirely arbitrary.
- I picked a sign convention for charge Q. With this convention $Q(t) > 0$ at any given time t means that at time $= t$ there is positive charge on the left side of the capacitor and $Q(t) < 0$ means the opposite. This choice is also entirely arbitrary.

Next I have to figure out the relationship between I and Q. Is it $I = +dQ/dt$ or $I = -dQ/dt$? With my convention, if I is positive then positive charge flows on the left side of the capacitor, so with my sign convention for Q, dQ/dt is also positive. Thus $I = +dQ/dt$.

Kirchoff's law is $(V_A - V_B) + (V_B - V_C) + (V_C - V_D) + (V_D - V_A) = 0$. This is not a fancy law, it is just elementary-school algebra **(but see the comment at the end!)**. Now we have to figure out all the $(V_i - V_j)'s$ one-by-one and get the signs right. Here it goes:

- $V_A - V_B = +IR$. Why? Because current in a resistor flows from the more positive end to the less positive end. With our convention if $I > 0$ current flows from A to B, and so $V_A > V_B$ if $I > 0$ Therefore I need a plus sign.
- $V_C - V_D = +Q/C$. Why? Because when positive charge is on one side of a capacitor then that side is at the higher voltage. With my charge convention for Q, when Q is positive $V_C > V_D$. So a plus sign.
- $V_D - V_A = -V$. Why? Here I am interpreting the circuit diagram notation as telling me that when V is positive the left side of the battery (or the AC generator) is more positive than the right side. The left side is node A, the right side is note D, so when V is positive $V_A > V_D$. So a negative sign.
- $V_B - V_C$ is the trickiest. Is it $+LdI/dt$ or $-LdI/dt$? Realize that $V_B - V_C$ is an emf so think of it as sticking a battery in the circuit instead of having the inductor. This battery (emf) wants to drive a current that opposes the change in I. So if dI/dt is positive, *i.e.*, if I increases in the clockwise direction, then this battery (emf) would drive current in the anticlockwise direction. A battery between nodes B and C would drive current in the anticlockwise direction if $V_B > V_C$. Thus $V_B - V_c = +LdI/dt$.

  Putting it all together Kirchoff's law becomes

$$IR + L\frac{dI}{dt} + \frac{Q}{C} - V = 0$$
$$R\frac{dQ}{dt} + L\frac{d^2Q}{dt^2} + \frac{Q}{C} = V$$

Now, to be perfectly honest, I cheated a little bit. Kirchoff's law for voltages is based on the assumption that $\oint \mathbf{E} \cdot \mathbf{dl} = 0$. This does not apply if they are changing magnetic fields. But in an inductor, if the current changes, *i.e.*, $\frac{dI}{dt} \neq 0$ the assumption fails. This is discussed in these two nice videos from Rice University. Take a few minutes to watch them

1. https://www.youtube.com/watch?v=ld6d_nRTIl4
2. https://www.youtube.com/watch?v=UNmEayHrCJg