

## VECTOR DERIVATIVES

## VECTOR IDENTITIES

**Cartesian.**  $d\mathbf{l} = dx \hat{\mathbf{x}} + dy \hat{\mathbf{y}} + dz \hat{\mathbf{z}}$ ;  $d\tau = dx dy dz$

$$\text{Gradient: } \nabla t = \frac{\partial t}{\partial x} \hat{\mathbf{x}} + \frac{\partial t}{\partial y} \hat{\mathbf{y}} + \frac{\partial t}{\partial z} \hat{\mathbf{z}}$$

$$\text{Divergence: } \nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\text{Curl: } \nabla \times \mathbf{v} = \left( \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{\mathbf{x}} + \left( \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{\mathbf{y}} + \left( \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{\mathbf{z}}$$

$$\text{Laplacian: } \nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

**Spherical.**  $d\mathbf{l} = dr \hat{\mathbf{r}} + r d\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi}$ ;  $d\tau = r^2 \sin \theta dr d\theta d\phi$

$$\text{Gradient: } \nabla t = \frac{\partial t}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial t}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial t}{\partial \phi} \hat{\phi}$$

$$\text{Divergence: } \nabla \cdot \mathbf{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta v_\theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi}$$

$$\begin{aligned} \text{Curl: } \nabla \times \mathbf{v} &= \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (\sin \theta v_\phi) - \frac{\partial v_\theta}{\partial \phi} \right] \hat{\mathbf{r}} \\ &\quad + \frac{1}{r} \left[ \frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r v_\phi) \right] \hat{\theta} + \frac{1}{r} \left[ \frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\phi} \end{aligned}$$

$$\text{Laplacian: } \nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 t}{\partial \phi^2}$$

**Cylindrical.**  $d\mathbf{l} = ds \hat{\mathbf{s}} + s d\phi \hat{\phi} + dz \hat{\mathbf{z}}$ ;  $d\tau = s ds d\phi dz$

$$\text{Gradient: } \nabla t = \frac{\partial t}{\partial s} \hat{\mathbf{s}} + \frac{1}{s} \frac{\partial t}{\partial \phi} \hat{\phi} + \frac{\partial t}{\partial z} \hat{\mathbf{z}}$$

$$\text{Divergence: } \nabla \cdot \mathbf{v} = \frac{1}{s} \frac{\partial}{\partial s} (sv_s) + \frac{1}{s} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\text{Curl: } \nabla \times \mathbf{v} = \left[ \frac{1}{s} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right] \hat{\mathbf{s}} + \left[ \frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s} \right] \hat{\phi} + \frac{1}{s} \left[ \frac{\partial}{\partial s} (sv_\phi) - \frac{\partial v_s}{\partial \phi} \right] \hat{\mathbf{z}}$$

$$\text{Laplacian: } \nabla^2 t = \frac{1}{s} \frac{\partial}{\partial s} \left( s \frac{\partial t}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

## Triple Products

$$(1) \quad \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

$$(2) \quad \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

## Product Rules

$$(3) \quad \nabla(fg) = f(\nabla g) + g(\nabla f)$$

$$(4) \quad \nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla) \mathbf{B} + (\mathbf{B} \cdot \nabla) \mathbf{A}$$

$$(5) \quad \nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$$

$$(6) \quad \nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

$$(7) \quad \nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$$

$$(8) \quad \nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla) \mathbf{A} - (\mathbf{A} \cdot \nabla) \mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$$

## Second Derivatives

$$(9) \quad \nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$(10) \quad \nabla \times (\nabla f) = 0$$

$$(11) \quad \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

## FUNDAMENTAL THEOREMS

$$\text{Gradient Theorem: } \int_{\mathbf{a}}^{\mathbf{b}} (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$$

$$\text{Divergence Theorem: } \int (\nabla \cdot \mathbf{A}) d\tau = \oint \mathbf{A} \cdot d\mathbf{a}$$

$$\text{Curl Theorem: } \int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$$

## BASIC EQUATIONS OF ELECTRODYNAMICS

## FUNDAMENTAL CONSTANTS

## Maxwell's Equations

*In general:*

$$\begin{cases} \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{cases}$$

*In matter:*

$$\begin{cases} \nabla \cdot \mathbf{D} = \rho_f \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t} \end{cases}$$

## Auxiliary Fields

*Definitions:*

$$\begin{cases} \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M} \end{cases}$$

## Potentials

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A}$$

## Lorentz force law

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

## Energy, Momentum, and Power

$$\text{Energy: } U = \frac{1}{2} \int \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) d\tau$$

$$\text{Momentum: } \mathbf{P} = \epsilon_0 \int (\mathbf{E} \times \mathbf{B}) d\tau$$

$$\text{Poynting vector: } \mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B})$$

$$\text{Larmor formula: } P = \frac{\mu_0}{6\pi c} q^2 a^2$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

(permittivity of free space)

$$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$$

(permeability of free space)

$$c = 3.00 \times 10^8 \text{ m/s}$$

(speed of light)

$$e = 1.60 \times 10^{-19} \text{ C}$$

(charge of the electron)

$$m = 9.11 \times 10^{-31} \text{ kg}$$

(mass of the electron)

## SPHERICAL AND CYLINDRICAL COORDINATES

## Spherical

$$\begin{cases} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{cases}$$

$$\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = \tan^{-1}(\sqrt{x^2 + y^2}/z) \\ \phi = \tan^{-1}(y/x) \end{cases}$$

$$\begin{cases} \hat{\mathbf{x}} = \sin \theta \cos \phi \hat{\mathbf{r}} + \cos \theta \cos \phi \hat{\mathbf{\theta}} - \sin \phi \hat{\mathbf{\phi}} \\ \hat{\mathbf{y}} = \sin \theta \sin \phi \hat{\mathbf{r}} + \cos \theta \sin \phi \hat{\mathbf{\theta}} + \cos \phi \hat{\mathbf{\phi}} \\ \hat{\mathbf{z}} = \cos \theta \hat{\mathbf{r}} - \sin \theta \hat{\mathbf{\theta}} \end{cases}$$

$$\begin{cases} \hat{\mathbf{r}} = \sin \theta \cos \phi \hat{\mathbf{x}} + \sin \theta \sin \phi \hat{\mathbf{y}} + \cos \theta \hat{\mathbf{z}} \\ \hat{\mathbf{\theta}} = \cos \theta \cos \phi \hat{\mathbf{x}} + \cos \theta \sin \phi \hat{\mathbf{y}} - \sin \theta \hat{\mathbf{z}} \\ \hat{\mathbf{\phi}} = -\sin \phi \hat{\mathbf{x}} + \cos \phi \hat{\mathbf{y}} \end{cases}$$

## Cylindrical

$$\begin{cases} x = s \cos \phi \\ y = s \sin \phi \\ z = z \end{cases}$$

$$\begin{cases} s = \sqrt{x^2 + y^2} \\ \phi = \tan^{-1}(y/x) \\ z = z \end{cases}$$

$$\begin{cases} \hat{\mathbf{x}} = \cos \phi \hat{\mathbf{s}} - \sin \phi \hat{\mathbf{\phi}} \\ \hat{\mathbf{y}} = \sin \phi \hat{\mathbf{s}} + \cos \phi \hat{\mathbf{\phi}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

$$\begin{cases} \hat{\mathbf{s}} = \cos \phi \hat{\mathbf{x}} + \sin \phi \hat{\mathbf{y}} \\ \hat{\mathbf{\phi}} = -\sin \phi \hat{\mathbf{x}} + \cos \phi \hat{\mathbf{y}} \\ \hat{\mathbf{z}} = \hat{\mathbf{z}} \end{cases}$$

Maxwell Equations in integral form:

$$\begin{aligned}\int_S \mathbf{D} \cdot d\mathbf{a} &= Q_{f\text{enclosed}} & \int_S \mathbf{B} \cdot d\mathbf{a} &= 0 \\ \int_P \mathbf{E} \cdot d\mathbf{l} &= -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{a} & \int_P \mathbf{H} \cdot d\mathbf{l} &= I_{\text{enclosed}} + \frac{d}{dt} \int_S \mathbf{D} \cdot d\mathbf{a}\end{aligned}$$

Boundary conditions at the interface of two materials.

$$\begin{aligned}D_1^\perp - D_2^\perp &= \sigma_f & B_1^\perp - B_2^\perp &= 0 \\ \mathbf{E}_1^\parallel - \mathbf{E}_2^\parallel &= 0 & \mathbf{H}_1^\parallel - \mathbf{H}_2^\parallel &= \mathbf{K}_f \times \hat{\mathbf{n}}\end{aligned}$$

Wave equation in vacuum:

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad \nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

Speed of light in vacuum  $c = 1/\sqrt{\epsilon_0 \mu_0}$ . In dielectric  $v = \frac{c}{n}$  where  $n = \sqrt{\epsilon \mu / \epsilon_0 \mu_0}$ .

Plane wave solution propagating in direction  $\hat{\mathbf{k}}$  (not in metals)

$$\mathbf{E} = \mathbf{E}_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t + \delta) \quad \mathbf{B} = \frac{1}{v} \hat{\mathbf{k}} \times \mathbf{E}$$

with  $\mathbf{E}_0$  perpendicular to  $\hat{\mathbf{k}}$  and  $v = \omega/k$ .

Snell's law:  $\frac{\sin \theta_T}{\sin \theta_I} = \frac{n_1}{n_2}$ .

For linearly polarized waves with polarization in the material boundary plane:

$$E_{0R} = \frac{\alpha - \beta}{\alpha + \beta} E_{0I} \quad E_{0T} = \frac{2}{\alpha + \beta} E_{0I} \quad \alpha = \frac{\cos \theta_T}{\cos \theta_I} \quad \beta = \frac{\mu_1 v_1}{\mu_2 v_2}$$

Wave equation in metal:

$$\nabla^2 \mathbf{E} = \mu \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu \sigma \frac{\partial \mathbf{E}}{\partial t} \quad \nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} + \mu \sigma \frac{\partial \mathbf{B}}{\partial t}$$

Solution for propagation in  $z$  direction:

$$\begin{aligned}\tilde{\mathbf{E}} &= \tilde{\mathbf{E}}_0 e^{-\kappa z} e^{i(kz - \omega t)} & \tilde{\mathbf{B}} &= \tilde{\mathbf{B}}_0 e^{-\kappa z} e^{i(kz - \omega t)} \\ k &= \omega \sqrt{\frac{\epsilon \mu}{2}} \left[ \sqrt{1 + \left( \frac{\sigma}{\epsilon \omega} \right)^2} + 1 \right]^{1/2} & \kappa &= \omega \sqrt{\frac{\epsilon \mu}{2}} \left[ \sqrt{1 + \left( \frac{\sigma}{\epsilon \omega} \right)^2} - 1 \right]^{1/2}\end{aligned}$$

Maxwell equations written in terms of potentials:

$$\begin{aligned}\nabla^2 V + \frac{\partial}{\partial t} (\nabla \cdot \mathbf{A}) &= -\frac{\rho}{\epsilon_0} \\ (\nabla^2 \mathbf{A} - \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{A}}{\partial t^2}) - \nabla (\nabla \cdot \mathbf{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t}) &= -\mu_0 \mathbf{J}\end{aligned}$$

Gauge transformations:  $\mathbf{A}' = \mathbf{A} + \nabla \lambda$  and  $V' = V - \frac{\partial \lambda}{\partial t}$ .

Lorenz gauge:  $\nabla \cdot \mathbf{A} = -\mu_0 \epsilon_0 \frac{\partial V}{\partial t}$     Coulomb gauge  $\nabla \cdot \mathbf{A} = 0$ .

In Lorenz gauge:  $\square^2 V = -\rho/\epsilon_0$      $\square^2 \mathbf{A} = -\mu_0 \mathbf{J}$

Retarded potentials:

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}', t_r)}{\mathbf{z}} d\tau' \quad \mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}', t_r)}{\mathbf{z}} d\tau'$$

Electric and magnetic fields (Jefimenko equations):

$$\begin{aligned} \mathbf{E}(\mathbf{r}, t) &= \frac{1}{4\pi\epsilon_0} \int \left[ \frac{\rho(\mathbf{r}', t_r)}{\mathbf{z}^2} \hat{\mathbf{z}} + \frac{\dot{\rho}(\mathbf{r}', t_r)}{c\mathbf{z}} \hat{\mathbf{z}} + \frac{\dot{\mathbf{J}}(\mathbf{r}', t_r)}{c^2\mathbf{z}} \right] d\tau' \\ \mathbf{B}(\mathbf{r}, t) &= \frac{\mu_0}{4\pi} \int \left[ \frac{\mathbf{J}(\mathbf{r}', t_r)}{\mathbf{z}^2} + \frac{\dot{\mathbf{J}}(\mathbf{r}', t_r)}{c\mathbf{z}} \right] \times \hat{\mathbf{z}} d\tau' \end{aligned}$$

Lienard-Wiechter potentials:

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{qc}{c\mathbf{z} - \mathbf{z} \cdot \mathbf{v}} \quad \mathbf{A}(\mathbf{r}, t) = \frac{V(\mathbf{r}, t)}{c^2} \mathbf{v}$$

Field of a moving point charge ( $\mathbf{u} \equiv c\hat{\mathbf{z}} - \mathbf{v}$ ):

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{\mathbf{z}}{(\mathbf{z} \cdot \mathbf{u})^3} [(c^2 - v^2)\mathbf{u} - \mathbf{z} \times (\mathbf{u} \times \mathbf{a})] \quad \mathbf{B}(\mathbf{r}, t) = \frac{1}{c} \hat{\mathbf{z}} \times \mathbf{E}(\mathbf{r}, t)$$

Electric dipole radiation, approximate formula. (Note:  $\ddot{\mathbf{p}}$  at retarded time):

$$\mathbf{E}(\mathbf{r}, t) = \frac{\mu_0}{4\pi r} [\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \ddot{\mathbf{p}})] \quad \mathbf{B}(\mathbf{r}, t) = -\frac{\mu_0}{4\pi rc} (\hat{\mathbf{r}} \times \ddot{\mathbf{p}})$$

Larmor formula:  $P = \frac{\mu_0 q^2 a^2}{6\pi c}$

Lienard formula:  $P = \frac{\mu_0 q^2 \gamma^6}{6\pi c} (a^2 - \frac{1}{c^2} |\mathbf{v} \times \mathbf{a}|^2)$

Boost with velocity  $v$  in  $x$  direction:

$$\begin{aligned} x' &= \gamma(x - \beta t) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - \beta x/c) \end{aligned}$$

Time-position and energy-momentum contravariant four vectors:

$$\begin{aligned} x^\mu &: (ct; \mathbf{r}) \\ p^\mu &: (E/c; \mathbf{p}) \end{aligned}$$

Contravariant ( $a^\mu$ ) to covariant ( $a_\mu$ ) relations:  $a_0 = -a^0$  and  $a_i = a^i$  for  $i = 1, 2, 3$ .

Lorentz invariant:  $a_\mu b_\mu$ . In particular  $p_\mu p^\mu = -E^2/c^2 + p^2$  leads to  $E^2 = m^2 c^4 + p^2 c^2$ .

Other important relationships:  $E = \gamma mc^2$  and  $\mathbf{p} = \gamma m\mathbf{v}$ .

Lorentz transformation matrix (boost velocity in  $x$  direction);

$$\Lambda_\nu^\mu = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Transformation law for four vectors:  $a^\mu \rightarrow \Lambda_\nu^\mu a^\nu$

Transformation law for EM fields:

$$\begin{aligned}\mathbf{E}_{\parallel} &\rightarrow \mathbf{E}_{\parallel} \\ \mathbf{B}_{\parallel} &\rightarrow \mathbf{B}_{\parallel} \\ \mathbf{E}_{\perp} &\rightarrow \gamma(\mathbf{E}_{\perp} + \mathbf{v} \times \mathbf{B}_{\perp}) \\ \mathbf{B}_{\perp} &\rightarrow \gamma(\mathbf{B}_{\perp} - \frac{\mathbf{v}}{c^2} \times \mathbf{E}_{\perp})\end{aligned}$$

EM Field tensor

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_z & 0 \end{bmatrix}$$

The (dual)  $G^{\mu\nu}$  tensor is obtained from the  $F^{\mu\nu}$  tensor by swapping  $\mathbf{E}/c$  with  $\mathbf{B}$  and  $\mathbf{B}$  with  $-\mathbf{E}/c$ .

The transformation law is  $F^{\mu\nu} \rightarrow \Lambda_{\alpha}^{\mu} \Lambda_{\beta}^{\nu} F^{\alpha\beta}$  and similarly for  $G^{\mu\nu}$ .

Covariant and contravariant partial derivatives:  $\partial_{\mu} = \frac{\partial}{\partial x^{\mu}}$  and  $\partial^{\mu} = \frac{\partial}{\partial x_{\mu}}$ .

Maxwell equations with  $J^{\mu} = (c\rho; \mathbf{J})$ :  $\partial_{\nu} F^{\mu\nu} = \mu_0 J^{\mu}$  and  $\partial_{\nu} G^{\mu\nu} = 0$ .

Continuity equation:  $\partial_{\mu} J^{\mu} = 0$

Four potential:  $A^{\mu} = (V/c; \mathbf{A})$ .

Field tensor from potential:  $F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$ .