HOMEWORK 6

GRIFFITHS 10.13

With $\vec{J}$ constant in time $\quad \vec{s} = \vec{r} - \vec{r}\,' \quad t_R = t - s/c$

$$\vec{E}(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int\left[\frac{\rho(\vec{r}\,',t_R)}{s^2} + \frac{\frac{\partial\rho}{\partial t}(\vec{r}\,',t_R)}{cs}\right]\hat{s}\,d\tau'$$

Since $\vec{J}$ constant in time $\quad \frac{\partial\rho}{\partial t}(\vec{r}\,',t_R) = \frac{\partial\rho}{\partial t}(\vec{r}\,',0)$

$$\frac{\partial\rho}{\partial t}(\vec{r}\,',t_R) = k$$

which then must mean that

$$\rho(\vec{r}\,',t_R) = \rho_0 + k t_R$$

and

$$\rho(\vec{r}\,',t) = \rho_0 + kt$$

where $\rho_0 = \rho(\vec{r}\,',0)$

This then gives

$$\vec{E}(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int\left[\frac{\rho_0 + k t_R}{s^2} + \frac{k}{cs}\right]\hat{s}\,d\tau'$$

But $t_R = t - s/c$

$$\vec{E}(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int\left[\frac{\rho_0 + kt - ks/c}{s^2} + \frac{k}{cs}\right]\hat{s}\,d\tau'$$

$$\vec{E}(\vec{r}',t) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_0 + kt}{s^2}\,\hat{s}\,d\tau'$$

and since we said $\rho(\vec{r}',t) = \rho_0 + kt$ this becomes

$$\boxed{\vec{E}(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}',t)}{s^2}\,\hat{s}\,d\tau'}$$

GRIFFITH 10.14

Again $\vec{s} = \vec{r} - \vec{r}'$

$t_R = t - s/c$

$$\vec{B} = \frac{\mu_0}{4\pi}\int\left[\frac{\vec{J}(\vec{r}',t_R)}{s^2} + \frac{\frac{d\vec{J}}{dt}(\vec{r}',t_R)}{cs}\right]\times\hat{s}\,d\tau'$$

Taylor expand, keep only first term

$$\vec{J}(\vec{r}\,',t_R) \cong \vec{J}(\vec{r}\,',t) + \frac{d\vec{J}}{dt}(\vec{r}\,',t)(t_R - t)$$

$$\vec{J}(\vec{r}\,',t_R) \cong \vec{J}(\vec{r}\,',t) + \frac{d\vec{J}}{dt}(\vec{r}\,',t)\left(-\frac{s}{c}\right)$$

$$\vec{J}(\vec{r}\,';t_R) \approx \vec{J}(\vec{r}\,',t) - \frac{s}{c}\frac{d\vec{J}}{dt}(\vec{r}\,',t) \qquad (1)$$

Taking derivative of this, neglecting 2nd derivatives, I get

$$\frac{d\vec{J}}{dt}(\vec{r}\,';t_R) \approx \frac{d\vec{J}}{dt}(\vec{r}\,',t) \qquad (2)$$

Putting equations (1) and (2) into the equation for $\vec{B}$ I get

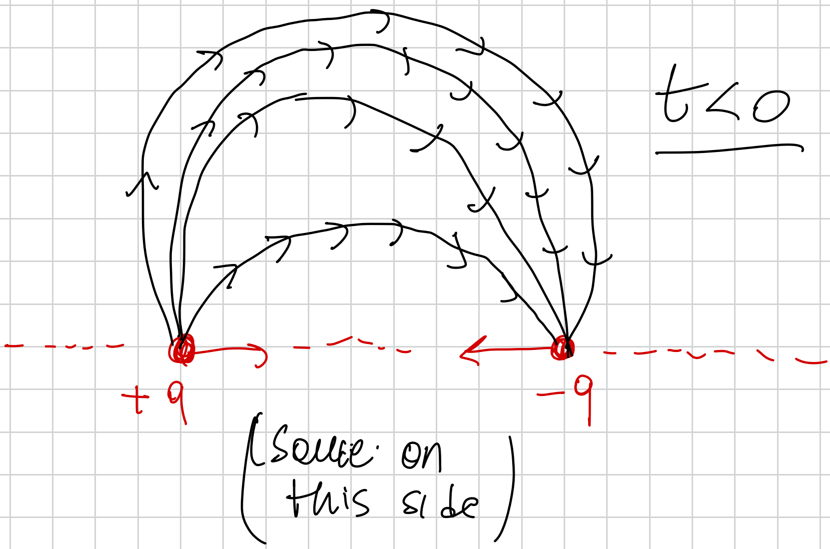
$$\vec{B} = \frac{\mu_0}{4\pi}\int\left[\frac{\vec{J}(\vec{r}\,',t) - \frac{s}{c}\frac{d\vec{J}}{dt}(\vec{r}\,';t)}{s^2} + \frac{\frac{d\vec{J}}{dt}(\vec{r}\,',t)}{cs}\right] \times \hat{s}$$

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,',t) \times \hat{s}}{s^2}}$$

GRIFFITHS 10.37

Imagine that collision happens at $t=0$

For $t<0$ we have field lines that look like a squished dipole. Squished because see Figure 10.10



After the collision, $t>0$ we have two distinct region. At a distance $r \leq ct$ from the origin, there will be no electric field.

At a distance $r \geq ct$ from the origin, the "news" that the two charges have neutralized each other has not arrived. IT WILL LOOK AS IF THE TWO CHARGES HAD CONTINUED THEIR TRAVEL

This means, looking back at my sketch as if $+q$ is on the left and $-q$ is on the right (VERY WEIRD) the direction of the $\vec{E}$ lines is REVERSED.

Since field lines are continous in regions of no charge, at or near $r = ct$ the field lines from the two sides join

At $r = ct$ the field lines are very close together. This is then a region of very high field that moves out at speed $= c$. Like an EM field pulse that is generated and moves o