PHYSICS 110B HOMEWORK 3

GRIFFITHS 7.57

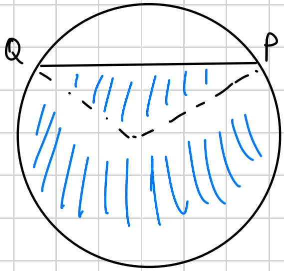
(a) $\text{emf} = -\dfrac{d\phi}{dt} = -\pi r^2 \dfrac{dB}{dt} = -\alpha \pi r^2$ since $B = \alpha t$ perpendicular to the loop

Then in magnitude

$$I = \frac{|\text{emf}|}{R}$$

$$\boxed{I = \frac{\alpha \pi r^2}{R} \text{ (magnitude)}}$$

(b)



Area of shaded area is $\frac{3}{4}$ the of the circle + the area of the triangle

$$A = \frac{3}{4}\pi r^2 + \frac{r^2}{2} = \frac{3\pi + 2}{4} r^2$$

The emf through the (PQP) circuit is

$$\text{emf}(PQP) = A\frac{dB}{dt} = \frac{3\pi+2}{4}\alpha r^2 \text{ in magnithe}$$

Kirchoff rule for PQP is $\text{emf}(PQP) = V + IR'$

where $R' =$ resistance between P and Q $= \frac{3}{4}R$

So $\quad V = \text{emf}(PQP) - IR' = \frac{3\pi+2}{4}\alpha r^2 - \frac{\alpha\pi r^2}{R}\frac{3}{4}R$

$$V = \alpha r^2 \frac{3\pi + 2 - 3\pi}{4}$$

$$\boxed{V = \frac{\alpha r^2}{2}}$$

$\boxed{\text{GRIFFITHS 8.15}}$ $\boxed{\text{Only a \& b}}$

(a) $\text{emf} = -\frac{d\phi}{dt} = \pi a^2 \frac{dB}{dt} \qquad B = \mu_0 n I_s \quad \frac{dB}{dt} = \mu_0 n \frac{dI_s}{dt}$

$\text{emf} = -\mu_0 n \pi a^2 \frac{dI_s}{dt} \qquad$ But $I_r = \text{emf}/R$

$$\Rightarrow \boxed{I_r = \frac{1}{R}\mu_0 n \pi a^2 \frac{dI_s}{dt} \text{ in magnitude}}$$

(b) I am going to use ρ as the distance from the axis, instead of s. Cylindrical coordinates $(\rho\phi z)$

Circular loop of radius ρ with $\rho > a$

$$\oint \bar{E}' \cdot \bar{dl}' = E_\phi 2\pi\rho = -\frac{d\phi}{dt} \Rightarrow \boxed{E_\phi = \frac{-\mu_0 a^2 n}{2\rho}\frac{dI_s}{dt}}$$

Note the $E_z = 0$ by symmetry (long solenoid)

Also $E_\rho = 0$ by taking Gauss law over a sphere centered on the axis.

Charges per unit length on the ring $= \lambda$

Drift velocity of charges $= v$ AND $I_r = \lambda v$

Force in $d\ell$: $dF = E_\phi \lambda d\ell = E_\phi \frac{I_r}{v} d\ell$

Power is $F \cdot v$. So power in $d\ell$ is $dP = dF \cdot v$

$$dP = E_\phi I_r d\ell$$

Integrating over the ring of radius b

$$P = 2\pi b \, E_\phi I_r = (2\pi b) \frac{\mu_0 a^2 n}{2b} \frac{dI_s}{dt} I_r$$

$$P = \pi \mu_0 a^2 n \frac{dI_s}{dt} I_r$$

But from part (e) $\frac{dI_s}{dt} = \frac{R I_r}{\mu_0 \pi a^2 n}$

Plugging this into the equation for power, $\boxed{P = I_r^2 R}$

$\boxed{\text{GRIFFITHS 9.2}}$

$f = A \sin kz \cos kvt$

$\frac{\partial f}{\partial z} = Ak \cos kz \cos kvt \quad \frac{\partial^2 f}{\partial z^2} = -Ak^2 \sin kz \cos kvt = -k^2 f$

$$\frac{\partial f}{\partial t} = -kv A \sin kz \sin kvt \qquad \frac{\partial^2 f}{\partial t^2} = (-k^2 v^2) A \sin kz \cos kvt$$

$$\frac{\partial^2 f}{\partial t^2} = -k^2 v^2 f$$

$$\Rightarrow \boxed{\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}}$$

Since $\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$ we can write

$f = A \sin kz \cos kvt$

$$\boxed{f = \frac{A}{2}\left[\sin k(z+vt) + \sin k(z-vt)\right]}$$

4

The centers of two circular metal plates forming a parallel plate capacitor, initially charged to voltage V_0, are connected internally by a straight fine wire of radius a, length L, and conductivity σ.
(a) Calculate the pointing vector (magnitude and direction) at the surface of the wire.
(b) Show that as the capacitor discharges from $V = V_0$ to $V = 0$ the total energy flowing into the wire is $E = \frac{1}{2}CV^2$, where C is the capacitance.
You can ignore the displacement current through the wire, which turns out to be a good approximation as long as the discharge of the capacitor is slow enough.

(a) The capacitor will discharge with a time constant

$\tau = RC \qquad R = \frac{L}{\sigma\pi a^2}$

$V = V_0 e^{-t/\tau}$ and the electric field is $E = \frac{V_0}{L} e^{-t/\tau}$

perpendicular to the plates and constant

The current density in the wire is $j = \sigma E$. So the

current through the wire is $I = \pi a^2 \sigma E$

On the surface of the wire the B field is tangential, and

$$\oint \vec{B}\,d\vec{l} = B\,2\pi a = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 \pi a^2 \sigma E}{2\pi a} = \frac{1}{2}\mu_0 a \sigma E$$

The Poynting vector is $\perp$ to $\vec{E}$ and $\perp$ to $\vec{B}$ so $\boxed{\text{Poynting vector into wire}}$

In magnitude the Poynting vector is $S = \frac{E \cdot B}{\mu_0}$

So

$$S = \frac{1}{2} a \sigma E^2 \quad \text{(using the expression for B)}$$

$$S = \frac{1}{2} a \sigma \frac{V_0^2}{L^2} \exp(-2t/RC) \qquad \text{But } R = L/\sigma\pi a^2$$

$$\boxed{S = \frac{1}{2} a \sigma \frac{V_0^2}{L^2} \exp\left(-\frac{2\sigma\pi a^2}{LC}\right)}$$

(b) To calculate the amount of energy flowing into the wire in dt, I need to multiply S by the surface area of the wire $(2\pi a L)$ and by dt

$$dE = (2\pi a L)\frac{1}{2} a \sigma E^2 dt = \pi a^2 L \sigma E^2 dt$$

$$\frac{dE}{dt} = \pi a^2 L \sigma \frac{V_0^2}{L^2} e^{-2t/RC} = \frac{\pi a^2 \sigma}{L} V_0^2 e^{-2t/RC}$$

$$\frac{dE}{dt} = \frac{1}{R} V_0^2 e^{-2t/RC} \qquad \text{since } R = \frac{L}{\sigma \pi a^2}$$

The total energy is $E = \int_0^\infty \frac{V_0^2}{R} e^{-2t/RC} dt$

$$\boxed{E = \frac{1}{2} V_0^2 / C}$$