PHYSICS 110B HOMEWORK 2

Griffiths 7.13

$$\Phi = B_0 \cos\omega t \, \pi \frac{a^2}{4}$$

$$\frac{d\Phi}{dt} = -\text{emf} = -\frac{a^2\pi}{4}\omega B_0 \sin\omega t$$

$$I = \frac{\text{emf}}{R}$$

$$\boxed{I = \frac{a^2\pi\omega B \sin\omega t}{4R}}$$

Griffiths 7.16

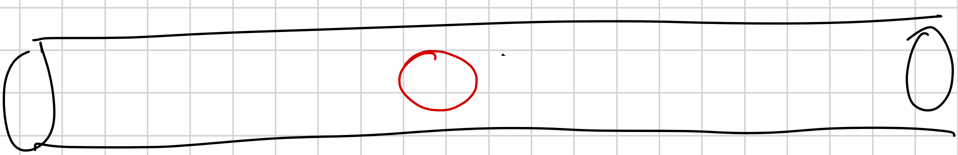
Inside : $B = \mu_0 N I \hat{z}$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu_0 N \frac{dI}{dt}\hat{z}$$

In cylindrical coordinates, by symmetry, and assuming infinite long solenoid, $E_z = 0$ and

$$\vec{E} = E_\phi(r)\,\hat{\phi} + E_r(r)\,\hat{r}$$

But $E_r(r)$ must be $= 0$ by considering the flux through the red spherical surface shown below



$\Phi = 4\pi r^2 E_r(r) = 0$ because no charge enclosed

Then $\vec{E} = E_\phi(r)\,\hat{\phi} = E(r)\,\hat{\phi}$

$$\left(\vec{\nabla}\times\vec{E}\right)_z = \frac{1}{r}\frac{\partial}{\partial r}(rE_\phi) = \frac{1}{r}\frac{\partial}{\partial r}(rE)$$

But $\vec{\nabla}\times\vec{E} = -\mu_0 N \dfrac{dI}{dt}$

$$\Rightarrow \quad \frac{\partial}{\partial r}(rE) = -\mu_0 N r \frac{dI}{dt}$$

$$rE = -\frac{\mu_0 N r^2}{2}\frac{dI}{dt}$$

$$\boxed{E = -\frac{\mu_0 N r}{2}\frac{dI}{dt}}$$ in $\hat{\phi}$ direction

Inside solenoid, $r < a$

Outside solenoid, $B=0$, therefore the same algebra leads to

$$\frac{\partial (rE)}{\partial r} = 0$$

which means that

$$rE = \text{Constant}$$

or $E = \dfrac{\text{constant}}{r}$

To insure continuity at $r=a$

$$\boxed{E = -\frac{\mu_0 N a^2}{2r}\frac{dI}{dt}}$$ in $\hat{\phi}$ direction

Outside solenoid, $r > a$

Another way of doing this

problem is to first convince yourself (as I did) that the only non zero component of the electric field is in the $\hat{\phi}$ direction and can only depend on r

Circular loop of radius r around the axis:

$$\oint \vec{E}\, d\vec{l} = 2\pi r E$$

and $$\oint \vec{E}\, d\vec{l} = -\frac{d\phi}{dt}$$

For $r < a$ $$\frac{d\phi}{dt} = \mu_0 N \pi r^2 \frac{dI}{dt}$$

and for $r > a$ $$\frac{d\phi}{dt} = \mu_0 N \pi a^2 \frac{dI}{dt}$$

this then allows you to solve for $\mathcal{E}$.

Griffiths 7.25

$B = \mu_0 n I$

Flux through one turn is

$\Phi = B\pi R^2 = \mu_0 n \pi R^2 I$

There are nl turns in a length l of solenoid

So the total flux for a length l is $\Phi = \mu_0 n^2 \pi R^2 l I$

If L is the self-inductance then $\Phi = LI$. Thus

$$\boxed{L = \mu_0 n^2 \pi R^2 \text{ per unit length}}$$

Griffiths 7.28

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

$$\frac{d^2Q}{dt} = -\frac{1}{LC}Q$$

This is the same eqtn as harmonic motion _ Undamped oscillator

$$Q(t) = Q_0\cos(\omega t + \delta)$$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

the $t=0$ condition gives

$\delta = 0$ and $Q_0 = CV$

$$I(t) = \frac{dQ}{dt}$$

$$\boxed{I(t) = -\omega CV \sin \omega t}$$

with a Resistor, the differential equation becomes

$$L\frac{d^2Q}{dt} - R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

This is the equation for a damped harmonic oscillator which I hope you have already seen in mechanics class (!)

$$\boxed{\text{Griffiths 7.47}}$$

(a) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ If $\vec{E} = 0$

$$\boxed{\frac{\partial \vec{B}}{\partial t} = 0}$$

(b) $\frac{d\phi}{dt} = -\oint \vec{E} \, d\vec{\ell}$

path through wire.

Since in wire $\vec{E} = 0$

$$\boxed{\frac{\partial \phi}{\partial t} = 0}$$

$$\boxed{\text{Griffiths 7.54}}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$$

Since the velocity is in the $\hat{y}$ direction we should switch to Cartesian coordinates

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\left(-\frac{y}{r}\hat{x} + \frac{x}{r}\hat{y}\right)$$

$$\vec{B} = \frac{\mu_0 I/2\pi}{x^2+y^2}\left(-y\hat{x} + x\hat{y}\right)$$

here x and y are measured from the wire. Since the wire is traveling to the right with velocity v I should replace y with $y - vt$ in the fixed coordinate system

$$\vec{B} = \frac{\mu_0 I/2\pi}{x^2+(y-vt)^2}\left[-(y-vt)\hat{x} + x\hat{y}\right)$$

Now I use $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\frac{\partial \vec{B}}{\partial t} = \frac{\mu_0 I/2\pi}{\left[x^2+(y-vt)^2\right]^2}\left[v\left(x^2+(y-vt)^2\right)\hat{x} + 2v(y-vt)\left(x\hat{y} - y\hat{x} + vt\hat{x}\right)\right]$$

At $t=0$

$$\frac{\partial \vec{B}}{\partial t} = \frac{\mu_0 I/2\pi}{(x^2+y^2)^2}\left[v(x^2+y^2)\hat{x} + 2vxy\,\hat{y} - 2vy^2\hat{x}\right]$$

$$\frac{\partial B}{\partial t} = \frac{\mu_0 I/2\pi}{(x^2+y^2)^2}\, v\left[(x^2-y^2)\hat{x} + 2xy\,\hat{y}\right]$$

At this point we should move back to cylindrical coordinates because at $t=0$ we have cylindrical symmetry

$$\hat{x} = \hat{r}\cos\phi - \sin\phi\,\hat{\phi}$$

$$\hat{y} = \hat{r}\sin\phi + \cos\phi\,\hat{\phi}$$

[Equations in book cover of Griffiths]

So the quantity in square brackets becomes

$$r^2\Big[(\cos^2\phi - \sin^2\phi)(\cos\phi\,\hat{r} - \sin\phi\,\hat{\phi}) + 2\sin\phi\cos\phi\,[\sin\phi\,\hat{r} + \cos\phi\,\hat{\phi})\Big]$$

$$= r^2\Big[(\cos^3\phi - \cos\phi\sin^2\phi + 2\sin^2\phi\cos\phi)\hat{r} + (\sin^3\phi - \sin\phi\cos^2\phi + 2\sin\phi\cos^2\phi)\hat{\phi}\Big]$$

$$= r^2\Big[\cos\phi\,[\cos^2\phi + \sin^2\phi]\,\hat{r} + \sin\phi\,[\sin^2\phi + \cos^2\phi]\,\hat{\phi}\Big]$$

$$= r^2\left(\cos\phi\,\hat{r} + \sin\phi\,\hat{\phi}\right)$$

Plugging this into the equation for $\frac{\partial B}{\partial t}$ I get

$$\frac{\partial \vec{B}}{\partial t} = \frac{\mu_0 I v}{2\pi r^2}\left[\cos\phi\,\hat{r} + \sin\phi\,\hat{\phi}\right]$$

Now I have to find $\vec{E}$ such that $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ and

$$\vec{\nabla}\vec{E} = 0$$

I note that $\vec{E}$ cannot depend on z by symmetry. Also, it needs to go to zero as $r \to \infty$

$$\vec{E}(r,\phi) = E_r(r,\phi)\,\hat{r} + E_\phi(r,\phi)\,\hat{\phi} + E_z(r,\phi)\,\hat{z}$$

$$\vec{\nabla}\cdot\vec{E} = 0 \Rightarrow \frac{1}{r}\frac{\partial}{\partial r}(rE_r) + \frac{1}{r}\frac{\partial E_\phi}{\partial \phi} = 0 \quad (1)$$

$$\left(\vec{\nabla}\times\vec{E}\right)_r = \frac{1}{r}\frac{\partial E_z}{\partial \phi} = -\frac{\mu_0 I v}{2\pi r^2}\cos\phi \quad (2)$$

$$\left(\vec{\nabla}\times\vec{E}\right)_\phi = -\frac{\partial E_z}{\partial r} = -\frac{\mu_0 I v}{2\pi r^2}\sin\phi \quad (3)$$

$$\left(\nabla\times E\right)_z = \frac{1}{r}\left[\frac{\partial}{\partial r}(rE_\phi) - \frac{\partial E_r}{\partial \phi}\right] = 0 \quad (4)$$

The 3rd equation gives

$$E_z = -\frac{\mu_0 I v}{2\pi r}\sin\phi$$

But since we want $\vec{E} \to 0$ as

$r \to \infty$, $f(\phi) = \text{constant} = 0$

So $$\boxed{E_z = -\frac{\mu_0 I v}{2\pi r} \sin\phi}$$

This solution also satisfies eqtn 2

Equations (1) and (4) are satisfied

if $$\boxed{E_r = E_\phi = 0}$$