Physics 110B HWK 1

Griffiths 7.2

(a) Kirchoff's law

$$\frac{Q}{C} = IR = \frac{dQ}{dt} R$$

$$\frac{dQ}{dt} - \frac{Q}{\tau} = 0 \qquad \tau \equiv RC$$

Solves as $Q = Q_0 e^{-t/\tau} = CV_0 e^{-t/\tau}$

$$\boxed{Q(t) = CV_0 e^{-t/\tau} \text{ with } \tau = RC}$$

$$I = -\frac{dQ}{dt} \Rightarrow \boxed{I(t) = \frac{V_0}{R} e^{-t/\tau}}$$

(b) $\boxed{E = \frac{1}{2} C V_0^2}$

$$\int_0^\infty P\,dt = \int_0^\infty I^2 R\,dt = \int_0^\infty \frac{V_0^2}{R} e^{-2t/\tau} dt$$

$$= \frac{V_0^2}{R}\left[-\frac{RC}{2} e^{-2t/RC}\right]_0^\infty$$

$$\boxed{\int_0^\infty P\,dt = \frac{1}{2} C V_0^2 = E}$$

(c) In part (a) we had $I = -\frac{dQ}{dt}$, ie decreasing the charge in the capacitor gave positive current. Now, it is the opposite ie, positive current corresponds to increased charge on the capacitor, ie $I = +\frac{dQ}{dt}$

Kirchoff law then $V_0 = QC + IR = QC + R\frac{dQ}{dt}$

This then gives

$$\frac{dQ}{dt} = \frac{1}{RC}\left[CV_0 - Q\right]$$ Remember $\tau \equiv RC$

$$\frac{dQ}{Q - CV_0} = -\frac{dt}{\tau}$$

Integrating both sides

$$\log(Q - CV_0) = -\frac{t}{\tau} + c_1$$ Some constant

$$Q = CV_0 + c_2 e^{-t/\tau} \qquad \left(c_2 = e^{c_1}\right)$$

At $t=0$, $Q=0$

this gives $c_2 = -CV_0$

Solution then is $$\boxed{Q(t) = CV_0\left[1 - e^{-t/\tau}\right]}$$

$$I(t) = \frac{dQ}{dt} = \frac{CV_0}{\tau} e^{-t/\tau}$$

and since $\tau = RC$ $$\boxed{I(t) = \frac{V_0}{R} e^{-t/\tau}}$$

(d) $\int_0^{\infty} V_0 I \, dt = \frac{V_0^2}{R} \int_0^{t} e^{-t/\tau} dt$

$= \frac{V_0^2}{R} \tau = \boxed{C V_0^2}$

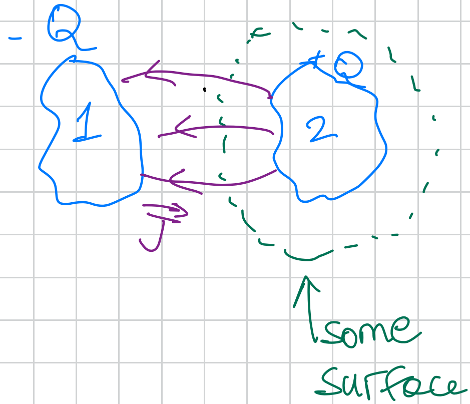
$I(t)$ is the same as in part (c) _

So once again, energy dissipated as heat in the resistor is $\frac{1}{2} C V_0^2$

At $t \to \infty$, $I = 0$ and voltage on capacitor is V_0, $\Rightarrow$ energy in capacitor is $\frac{1}{2} C V_0^2$

One half of the energy provided by the battery is dissipated as heat _ The other half is stored in the capacitor

Griffiths 7.3



(a)

Current flowing out of 2

$$I = \int \vec{J}\, d\vec{a}$$

over a surface enclosing 2.

But $\vec{J} = \sigma \vec{E}$

$$I = \sigma \int \vec{E}\, d\vec{a} = \sigma \frac{Q}{\varepsilon_0}$$ (Using Gauss' law)

But $Q = CV$ so $I = \dfrac{\sigma C V}{\varepsilon_0}$

or $V = I \dfrac{\varepsilon_0}{\sigma C}$. But $V = IR$

Therefore $\boxed{R = \dfrac{\varepsilon_0}{\sigma C}}$

(b) This is same as the previous problem. Everything happens with a time constant

$\tau = RC$ ie $\boxed{\tau = \varepsilon_0/\sigma}$

Griffiths 7.7

(a) $\frac{d\phi}{dt} = Bl v = \text{emf}$ (up to a sign that I don't care about right now)

$I = \frac{\text{emf}}{R}$ $\boxed{I = Blv/R}$

The flux into the page is increasing. The current wants to setup a B-field that keeps the flux as constant as possible, i.e. a B field out of the page. By the right-hand rule the induced current is counterclockwise through the resistor

(b) $F = IlB$ $\boxed{F = B^2l^2v/R}$

In the bar the current is flowing upwards

Then the force is to the left

(c) $F = ma$, with the acceleration being to the left, ie, in the direction opposite to the velocity

Acceleration $\frac{dv}{dt} = \frac{F}{m} = -\frac{B^2 \ell^2 v}{mR}$

(The minus sign is there because of this)

The simple differential equation has solution

$$\boxed{v(t) = v_0 \exp\left(-\frac{B^2 \ell^2 t}{mR}\right)}$$

(d) $E = \int_0^\infty I^2 R \, dt = \int_0^\infty \left(\frac{B \ell v}{R}\right)^2 R \, dt$

$$E = \int_0^\infty \frac{B^2 \ell^2 v^2}{R} dt = \frac{B^2 \ell^2 v_0^2}{R} \int_0^\infty \exp\left(-\frac{2B^2 \ell^2 t}{mR}\right) dt$$

$$E = \frac{B^2 \ell^2 v_0^2}{R} \cdot \frac{mR}{2B^2 \ell^2}$$

$$\boxed{E = \frac{1}{2} m v_0^2}$$

Griffiths 7.9

$\vec{\nabla}\vec{B}=0$ This then implies that the integral $\int \vec{B}\,d\vec{a}$ is the same through any surface, with any shape, with the same boundary line. See theorem 2c on page 52.

Griffiths 7.10

Let α be the angle that the perpendicular to the loop makes it with the B field

$\alpha = \omega t + \alpha_0$

Area of loop $A = a^2$

Flux $\Phi = BA\sin\phi = Ba^2\sin(\omega t+\alpha_0)$

$\text{emf} = -\frac{d\phi}{dt}$

$$\boxed{\text{emf} = -\omega B a^2 \cos \omega t + \alpha_0}$$