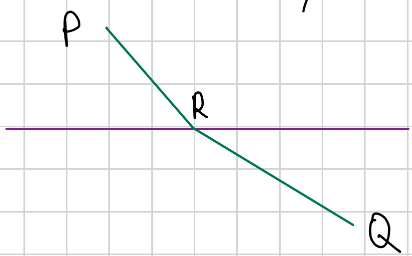
SESSION 5

Within a medium, the path is straight.



$P(x_1 \; 0 \; z_1)$

$R(x \; y \; 0)$

$Q(x_2 \; 0 \; z_2)$

To show that everything happens in a plane I need to show that $y=0$

$$L = n_1\left((x_1-x)^2 + y^2 + z_1^2\right)^{1/2} + n_2\left((x_2-x)^2 + y^2 + z_2^2\right)$$

I want $\dfrac{\partial L}{\partial x} = \dfrac{\partial L}{\partial y} = 0$

$$\frac{\partial L}{\partial y} = 2y\left(\frac{n_1}{d_1} + \frac{n_2}{d_2}\right)$$

where $d_1^2 = (x_1-x)^2 + y^2 + z_1^2$

$d_2^2 = (x_2-x)^2 + y^2 + z_2^2$

$\dfrac{\partial L}{\partial y} = 0 \Rightarrow \boxed{y=0}$ The segments PR and RQ are in a plane

For the second part, define incident and transmitted angles as the usual incidence and transmitted angles, ie, the angles that PR and RQ make with the vertical.

We then have that $|x - x_1| = r_1 \sin\theta_I$ and $|x_2 - x| = r_2 \sin\theta_R$.

Note: we must have either $x_1 < x < x_2$ or $x_1 > x > x_2$, obviously.

$$\frac{\partial \mathcal{L}}{\partial x} = n_1 \frac{x - x_1}{r} + n_2 \frac{x - x_2}{r}$$

$$\frac{\partial \mathcal{L}}{\partial r} = 0 \Rightarrow n_1 \sin\theta_I - n_2 \sin\theta_R = 0$$

The minus sign comes from the fact that if

$x - x_1 > 0$ $\quad x - x_2 < 0$ and viceversa.

This then gives SNELL'S LAW

$$\boxed{\frac{\sin\theta_I}{\sin\theta_R} = \frac{n_2}{n_1}}$$