# Complement to the lecture of October 22nd

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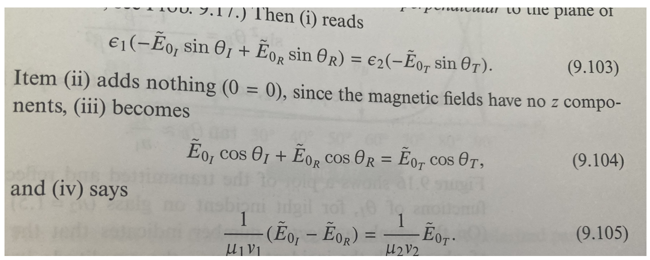
After the lecture some students were confused about the signs in the derivations of equations 9.103, 9.104, and 9.105.



9.17.) Then (i) reads

$$\epsilon_1(-\tilde{E}_{0_I}\sin\theta_I + \tilde{E}_{0_R}\sin\theta_R) = \epsilon_2(-\tilde{E}_{0_T}\sin\theta_T). \qquad (9.103)$$

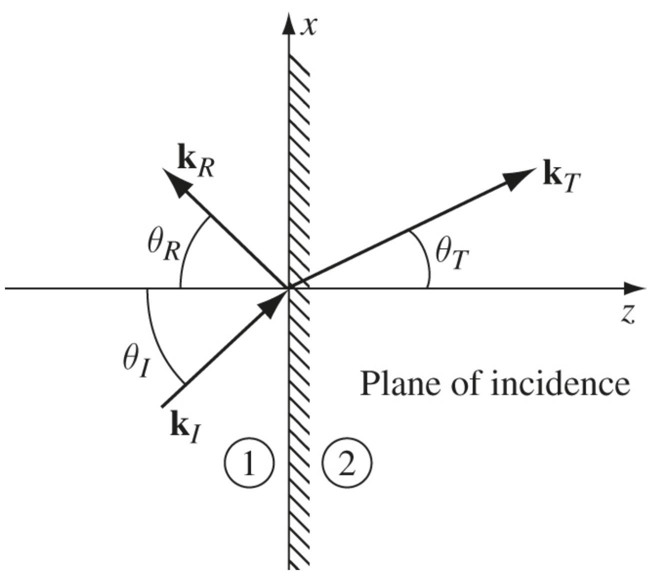
Item (ii) adds nothing (0 = 0), since the magnetic fields have no $z$ components, (iii) becomes

$$\tilde{E}_{0_I}\cos\theta_I + \tilde{E}_{0_R}\cos\theta_R = \tilde{E}_{0_T}\cos\theta_T, \qquad (9.104)$$

and (iv) says

$$\frac{1}{\mu_1 v_1}(\tilde{E}_{0_I} - \tilde{E}_{0_R}) = \frac{1}{\mu_2 v_2}\tilde{E}_{0_T}. \qquad (9.105)$$

Here is a more detailed explanation which, to make things simpler, does not use complex numbers. The starting point is Figure 9.14 in Griffiths:




Let's write unit vectors in the direction of propagation of these waves

$$\hat{k}_I = \sin\theta_I \hat{x} + \cos\theta_I \hat{z}$$
$$\hat{k}_R = \sin\theta_R \hat{x} - \cos\theta_R \hat{z}$$
$$\hat{k}_T = \sin\theta_T \hat{x} + \cos\theta_T \hat{z}$$

We are working with the polarization *i.e.*, the direction of the electric field, chosen to be in the $(xz)$ plane. The $\vec{E}$ fields have to be perpendicular to the $\hat{k}$ vectors, thus:

$$\vec{E}_I = E_{0I}(\cos\theta_I \hat{x} - \sin\theta_I \hat{z}) \cos(\vec{k}_I \vec{r} - \omega t)$$
$$\vec{E}_R = E_{0R}(\cos\theta_R \hat{x} + \sin\theta_R \hat{z}) \cos(\vec{k}_R \vec{r} - \omega t)$$
$$\vec{E}_T = E_{0T}(\cos\theta_T \hat{x} + \sin\theta_T \hat{z}) \cos(\vec{k}_T \vec{r} - \omega t)$$

Here the $E_{OJ}$ $(J = I, R, T)$ are just scalars (numbers in units of V/meter).

The magnetic fields are given by $\vec{B} = \frac{1}{v}(\hat{k} \times \vec{E})$, where $v$ is the velocity of the EM wave. Denoting the velocities by $v_1$ and $v_2$ to the left and the right of the boundary between the media, and taking cross products, we get

$$\vec{B}_I = \frac{E_{0I}}{v_1} \cos(\vec{k}_I \vec{r} - \omega t)\hat{y}$$
$$\vec{B}_R = -\frac{E_{0R}}{v_1} \cos(\vec{k}_R \vec{r} - \omega t)\hat{y}$$
$$\vec{B}_T = \frac{E_{0T}}{v_2} \cos(\vec{k}_T \vec{r} - \omega t)\hat{y}$$

Now we are ready to apply the boundary conditions on the surface. As argued in lecture, and also explained in Griffiths, on the surface the quantity $(\vec{k}_J \vec{r} - \omega t)$, where $J = I, R, T$, is the same for all $J$. So to implement the boundary conditions we can drop the cosine terms. The boundary conditions are

$$\left.\begin{array}{ll}
\text{(i)} & \epsilon_1(\tilde{\mathbf{E}}_{0_I} + \tilde{\mathbf{E}}_{0_R})_z = \epsilon_2(\tilde{\mathbf{E}}_{0_T})_z, \\
\text{(ii)} & (\tilde{\mathbf{B}}_{0_I} + \tilde{\mathbf{B}}_{0_R})_z = (\tilde{\mathbf{B}}_{0_T})_z, \\
\text{(iii)} & (\tilde{\mathbf{E}}_{0_I} + \tilde{\mathbf{E}}_{0_R})_{x,y} = (\tilde{\mathbf{E}}_{0_T})_{x,y}, \\
\text{(iv)} & \frac{1}{\mu_1}(\tilde{\mathbf{B}}_{0_I} + \tilde{\mathbf{B}}_{0_R})_{x,y} = \frac{1}{\mu_2}(\tilde{\mathbf{B}}_{0_T})_{x,y},
\end{array}\right\}$$

Equation (i) is for the $z$ components of the electric fields. This yields

$$\epsilon_1(-E_{0I}\sin\theta_I + E_{OR}\sin\theta_R = -\epsilon_2 E_{0T}\sin\theta_T$$

(same as equation 9.103)

Equation (iii) is for the $x$ and $y$ components of the electric field. The $y$ components are zero, so we are left with the $x$ component equation:

$$E_{0I}\cos\theta_I + E_{OR}\cos\theta_R) = E_{0T}\cos\theta_T$$

(same as equation 9.104)

Similarly, equation (iv) is for the $x$ and $y$ components of the magnetic field. The $x$ components are zero, so we are left with the $y$ component equation:

$$\frac{1}{\mu_1 v_1}(E_{0I} - E_{OR}) = \frac{1}{\mu_2 v_2}E_{0T}$$

(same as equation 9.105)