Problem 1
Four masses are placed in the (x,y) plane as follows:
(i) mass=2m, coordinates (a, b)
(ii) mass=m, coordinates (-a, b)
(iii) mass=2m, coordinates (-a, -b)
(iv) mass=m, coordinates (a, -b)

(a) Find the inertia tensor
(b) Now set a-b (for simplicity) and find the three principal moments of inertia
(c) Without doing any algebra, can you use your intuition about principal axes of inertia to indicate (for example, in a sketch) the directions of the three principal axes?

Problem 2
A uniform spherical planet of radius R rotates with constant angular velocity $\omega$. The effective gravitational acceleration $g_{\text{eff}}$ is some constant $g$ at the poles and 0.8g at the equator. Find the mass of the planet in terms of $\omega$, R, and the gravitational constant G.

Problem 3
A yo-yo of mass M is composed of two identical uniform disks of radius R held together through their centers by a massless shaft of radius r. A massless string is wound in the middle of the shaft, and the loose end is held in somebody’s hand. Upon release, the yo-yo descends until the string is unwound. Find the tension T of the string as the yo-yo is descending.

You can use the fact that the moment of inertia of a uniform disk of mass $\mu$ and radius $a$ for rotation around an axis that passes through the center of the disk and perpendicular to the plane of the disk is $I = \frac{1}{2} \mu a^2$. Don’t forget that in this problem M is the mass of the yo-yo, i.e., the combined mass of the two disks.

Problem 4
A bead of mass m is constrained to move without friction on a hoop of radius R. The hoop rotates with constant angular velocity $\omega$ about a vertical axis which coincides with a diameter of the hoop, see figure below

(a) By applying Newton’s laws, including fictitious forces, in the rest frame of the hoop, show that
$$\frac{d^2\theta}{dt^2} + \frac{(g/R - \omega^2 \cos\theta) \sin\theta}{\omega^2} = 0$$

If you are unable to derive this result, you can still use it, and proceed with the remainder of the problem
Hint: consider forces both perpendicular and tangent to the hoop
(b) Find the critical angular velocity $\Omega$ below which the bottom of the hoop provides a position of stable equilibrium for the hoop
(c) Find the equilibrium position for $\omega > \Omega$. 
Problem 5
A bug of mass \( m \) crawls out without slipping along the radius of a turntable rotating with angular velocity \( \omega \). If the bug is at a distance \( d \) from the center and it is traveling with velocity \( v \), what is the magnitude of the total force on the bug.

Problem 6
A body moves in an elliptical orbit under the influence of an attractive inverse square force (\( F=-k/r^2 \)). The major axis of the ellipse is 2a, the minor axis is 2b, and the eccentricity is \( \varepsilon \). (Note: \( b^2 = (1-\varepsilon^2)a^2 \)).
(a) sketch the ellipse and indicate where the velocity of the body will be minimum and maximum (justify!)
(b) show that \( V_{\text{max}}/V_{\text{min}} = (1+\varepsilon)/(1-\varepsilon) \), where \( V_{\text{max}} \) and \( V_{\text{min}} \) are the maximum and minimum values of the velocity.