Constrained fit exercise

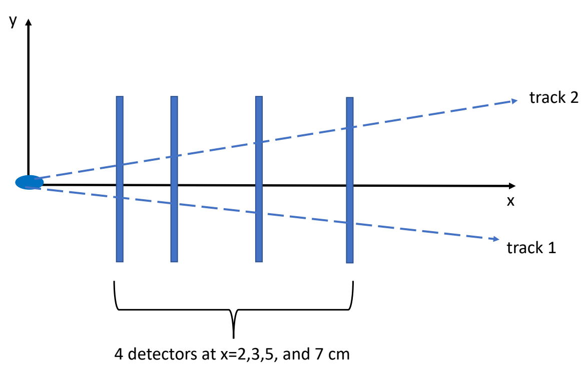
The experimental apparatus in the figure consists of four thin charged particle detectors at $x = 2, 3, 5$ and 7 cm. Each detector is segmented with many 50 μm wide strips that measure the y-coordinates of charged particles going through. (Note: if the width of the strip is w, the y-coordinate is measured with an uncertainty $w/\sqrt{12}$ [1]).

Particles traveling along the y-axis in the positive x direction decay somewhere in the vicinity of ($x = 0$ $y = 0$) into two charged particles that leave hits in the detectors. You are interested in finding the x coordinate of the decay for a given pair of tracks. A simulation of many such particle decays has been carried out. The simulated data are in a text file `https://tinyurl.com/n37jrpja`. Each line of this file has information for one pair of tracks as follows:

```
X0 Y0 y00 y01 y02 y03 y10 y11 y12 y13
where
X0, Y0 = true common point of origin of pair (Y0 always 0)
yij = digitized coordinate of track i at detector j
All lengths in cm
```

Write a constrained fit to estimate `X0` on an event by event basis, assuming that you know that `Y0=0`.

Plot the pull and the probability of chi-squared.



Note: this is a semi-realistic problem in HEX. In the real world, the tracks are

[1] If you do not believe this, calculate for yourself the variance of a set of numbers picked uniformly at random betwee 0 and w.

in 3D (not 2D), they bend in a magnetic field, they are constrained to come from the same 3D point (not simply $y = 0$), there may be more than 2 tracks in the vertex, the detector is more complicated, etc. etc. However, typically vertex fits do not take as inputs the detector hits. Instead, the detector hits are used to fit the tracks, and then the tracks, with their covariance matrices, are fed to the constrained fit machinery.