In class I showed an example Minuit fit (html and ipynb links) of μS+B with three bins with 20% lognormal and uncorrelated uncertainties on B and

- Data = 12 7 4
- B = 10 5 2
- S = 2 2 2

The fitted value was $\mu = 1.066^{+0.826}_{-0.696}$

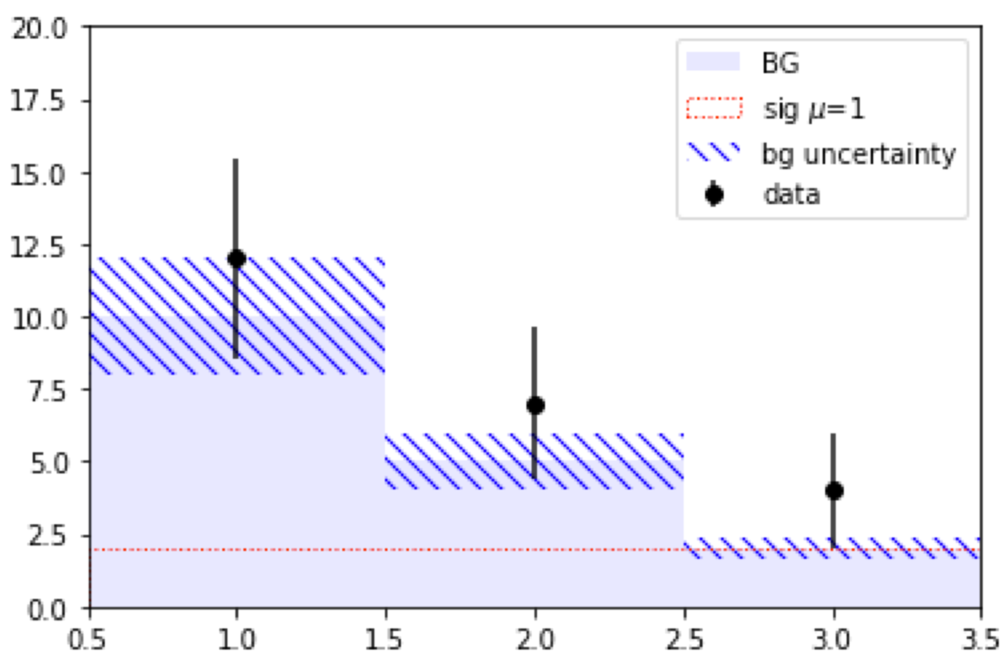
Since I (coincidentally?) chose Data=S+B Matthew was surprised that $\mu \neq 1.000$

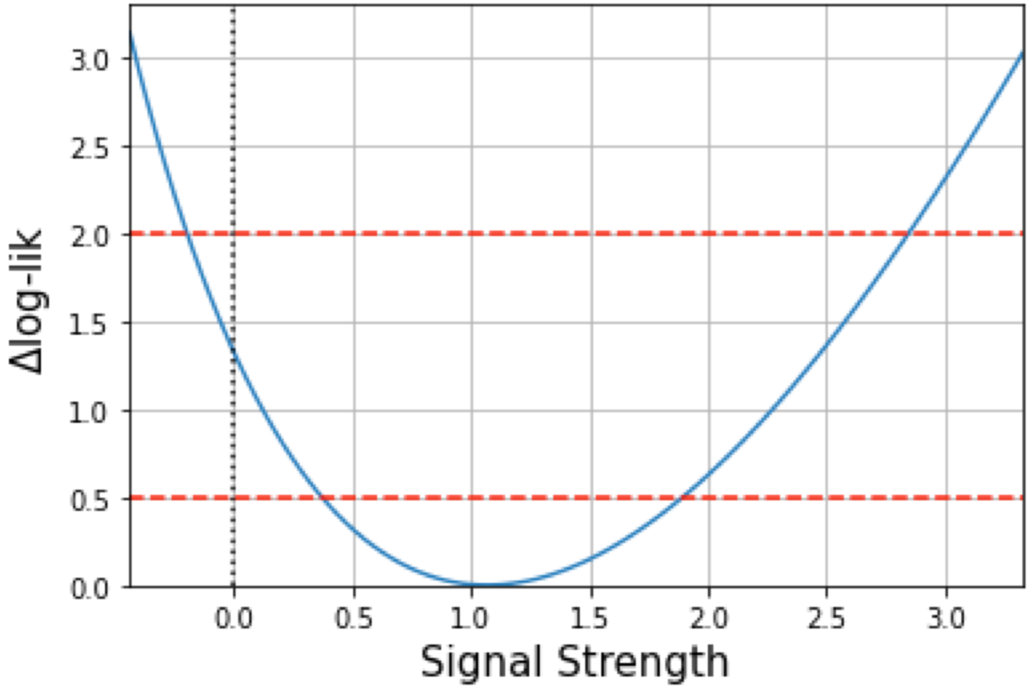
I blamed it on being in the Poisson regime.

That is not true.

Scale everything (Data, B, S) by x100, move into the Gaussian regime, fit returns

$$\mu = 1.045^{+0.173}_{-0.190}$$





The real reason

- Compare gaussian of mean$=10$ $\sigma=2$ with “equivalent” lognormal
- Lognormal is asymmetric
  - Mode   < 10
  - Median = 10
  - Mean   > 10

```
b, e = 10, 2
x  = np.linspace(0,20.,401)
y1 = lognorm.pdf(x, np.log(1+e/b), 0, b)
y2 = norm.pdf(x, scale=e, loc=b)

mean1, mean2 = (x*y1).sum() / y1.sum(), (x*y2).sum() / y2.sum()

z1, z2 = y1.cumsum() / y1.cumsum()[-1], y2.cumsum() / y2.cumsum()[-1]
z1, z2 = (z1 >= 0.5), (z2 >= 0.5)
median1, median2 = x[np.nonzero(z1==1)[0][0]], x[np.nonzero(z2==1)[0][0]]

i1, i2         = np.argmax(y1), np.argmax(y2)
mode1, mode2 = x[i1], x[i2]

print('lognormal mean   ~',  mean1)
print('lognormal mode   ~ ', mode1)
print('lognormal median ~ ', median1)
print('gaussian  mean   ~',  mean2)
print('gaussian  median ~ ', median2)
print('gaussian  mode   ~ ', mode2)
```

```
lognormal mean   ~ 10.166843497021965
lognormal mode   ~  9.65
lognormal median ~  10.0
gaussian  mean   ~ 10.000000000000002
gaussian  median ~  10.0
gaussian  mode   ~  10.0
```

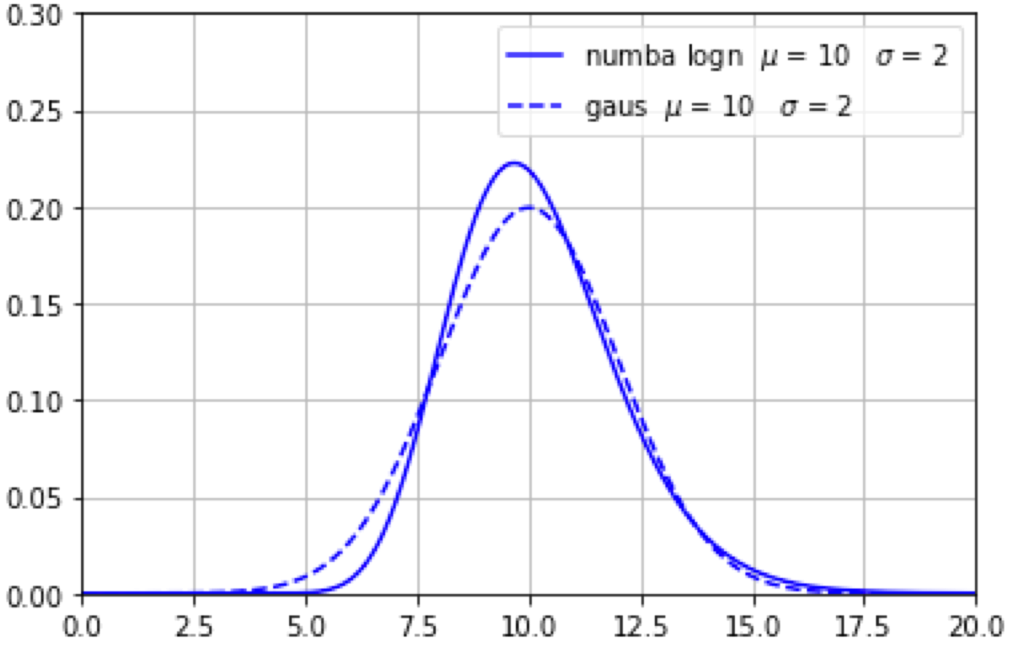
- Repeat fit using gaussian pdfs:

$$\mu = 1.000^{+0.830}_{-0.699}$$

- Also with x100 the stats, ie away from Poisson regime:

$$\mu = 1.000^{+0.201}_{-0.201}$$

  - With gauss pdf errors are now symmetric
  - Wasn’t the case with lognormal, see prev. page



Further Comment

- In the x100 statistics case (at right) the details on the choice of background uncertainty make a noticeable difference

- Lognormal: $\mu = 1.045^{+0.173}_{-0.190}$
- Gaussian: $\mu = 1.000^{+0.201}_{-0.201}$

- About ¼ of a σ shift in μ
- About 15% change in relative error σ/μ