Lognormal

The lognormal pdf for a variable $y > 0$ is:

$$p(y)dy = \tfrac{1}{\sqrt{2\pi}s}\ \tfrac{1}{y}\ \exp(-\tfrac{\log^2 y}{2s^2})\ dy$$

Some properties of the lognormal:

1. It is the pdf for the product of many positive random variables, just as the normal is the pdf for the sum of random variables.
2. The median (not the mean) is $y = 1$
3. $\lim_{y\to 0} p(y) = 0$
4. $w = \log y$ is a gaussian of mean 0 and $\sigma = s$
5. Substituting $y = x/\mu$ and $s = \log k$ with $k = 1 + \sigma/\mu$ yields a distribution for x which is approximately gaussian of mean μ and standard deviation σ

Proof of 3
Take $z = \log y$, i.e., $y = e^z$. Then

$$\lim_{y\to 0} p(y) = \lim_{z\to -\infty} p(z)$$

$$p(z) = \frac{1}{\sqrt{2\pi}s}\ e^{-z}\ e^{-z^2/(2s^2)} = \frac{1}{\sqrt{2\pi}s}\ e^{-z-z^2/(2s^2)}$$

as $z \to -\infty$, $e^{-z-z^2/(2s^2)} \to e^{-z^2/(2s^2)} \to 0$.

Proof of 4
$dw = dy/y$, therefore $p(w)dw = \frac{1}{\sqrt{2\pi}s} e^{-w^2/(2s^2)} dw$.

Proof of 5
Note $dy = dx/\mu$ so $dy/y = (dx/\mu)/(x/\mu) = dx/x$. Therefore:

$$p(x)dx = \frac{1}{\sqrt{2\pi}\log k}\ \frac{1}{x}\ \exp(-\frac{\log^2 x/\mu}{2\log^2 k})\ dx$$

Take x/μ close to 1 and $\sigma << \mu$.
Then $\log x/\mu = \log(1 + (x-\mu)/\mu) \approx (x-\mu)/\mu$. Similarly, $\log k \approx \sigma/\mu$.

$$p(x)dx \approx \frac{1}{\sqrt{2\pi}\sigma}\ \frac{\mu}{x}\ e^{\frac{-(x-\mu)^2}{2\sigma^2}}\ dx$$

and since $\mu/x \approx x/\mu \approx 1$, this is just a gaussian for x.