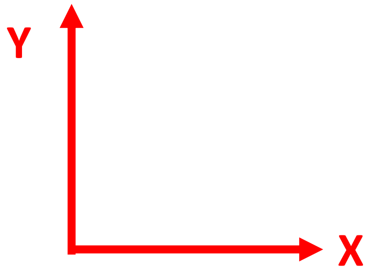
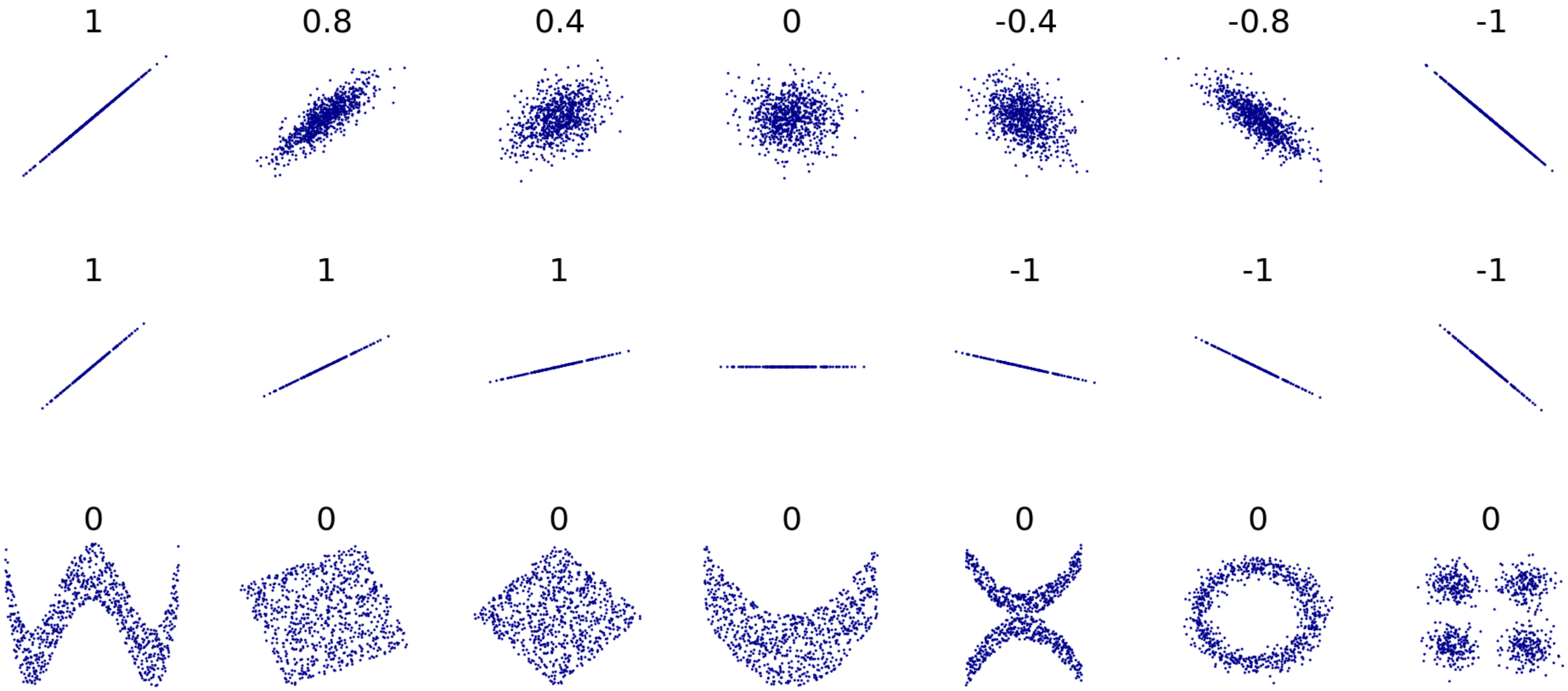
Pearson's correlation

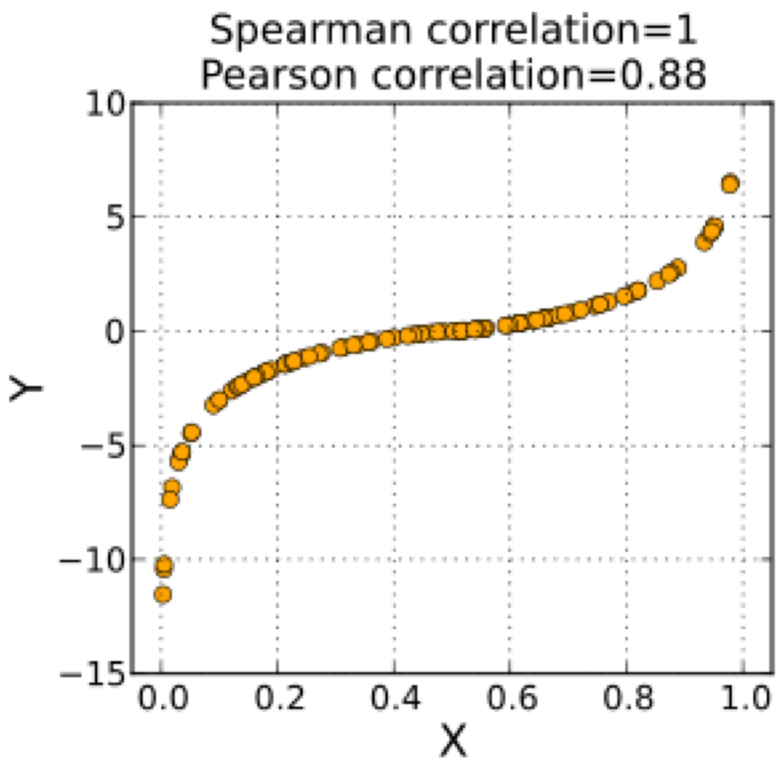


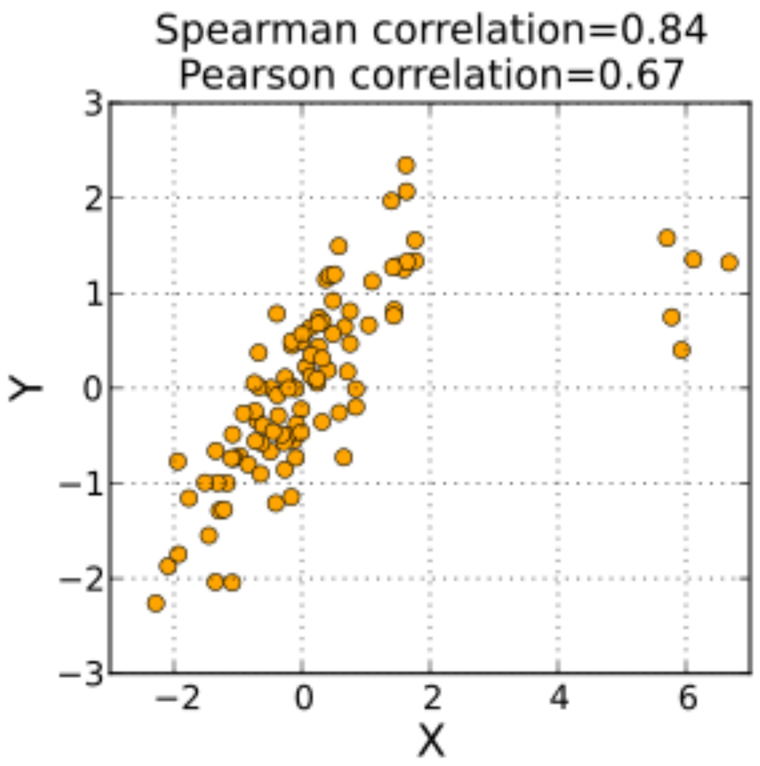
$$\rho_{X,Y} = \mathrm{corr}(X,Y) = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{\mathrm{E}[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$



https://en.wikipedia.org/wiki/Correlation

Spearman ρ





The Spearman correlation coefficient is defined as the Pearson correlation coefficient between the rank variables.[3]

For a sample of size *n*, the *n* raw scores X_i, Y_i are converted to ranks $\mathrm{R}(X_i), \mathrm{R}(Y_i)$, and r_s is computed as

$$r_s = \rho_{\mathrm{R}(X),\mathrm{R}(Y)} = \frac{\mathrm{cov}(\mathrm{R}(X), \mathrm{R}(Y))}{\sigma_{\mathrm{R}(X)}\sigma_{\mathrm{R}(Y)}},$$

https://en.wikipedia.org/wiki/Spearman%27s_rank_correlation_coefficient

Another measure is "Kendall τ"

Beam constrained mass M_{bc} aka energy substituted mass M_{ES}

$e^+e^- \to \Upsilon(4S) \to B\,\bar{B}$
Mass(Υ) = 10.578 GeV
2*Mass(B) = 10.558 GeV
In CM frame (which is known),

$$E_{expected}\,(B) = \frac{1}{2} Mass(\Upsilon)$$

Thus for a fully reconstructed decay there are two pieces of information:

1. The invariant mass
2. The energy

To (almost) decorrelate the two, B reconstruction is characterized by

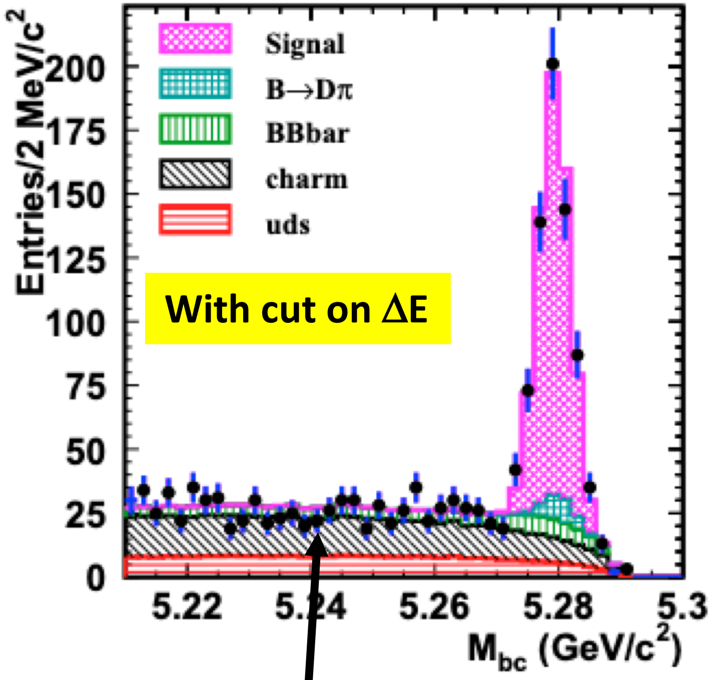
1. $M_{bc}^2 = E_{expected}^2 - P_{measured}^2\,(B)$
2. $\Delta E = E_{measured}(B) - E_{expected}(B)$

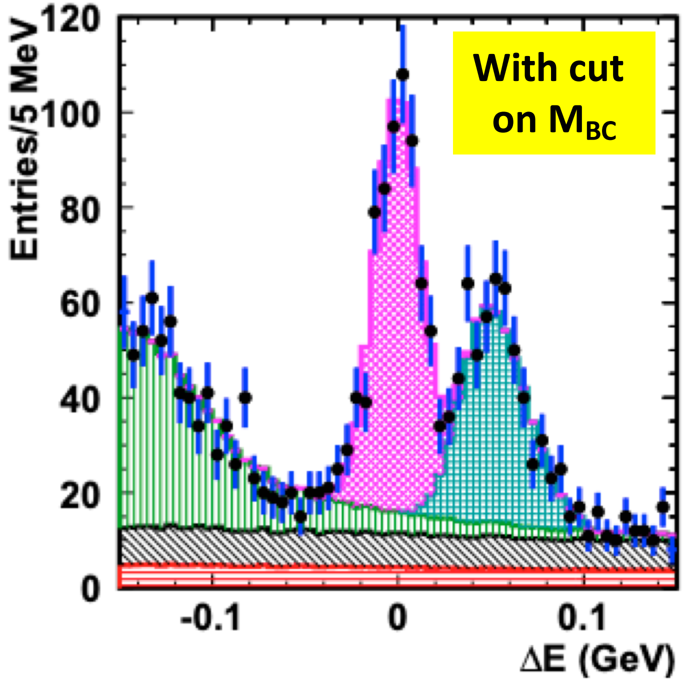
$P_{measured}^2$ and therefore M_{BC} does not depend on the particle ID, π vs. μ vs. K. But ΔE does.

The momentum of the B is very small

$$P(B) = \sqrt{\frac{1}{4}\,Mass^2(\Upsilon) - Mass^2(B)} = 325 \text{ MeV}$$

$B \to D^0(\to K_s\pi^+\pi^-)K^+$ vs. $B \to D^0(\to K_s\pi^+\pi^-)\,\pi^+$





Threshold (Argus) function

$$\sim x\sqrt{1 - x^2}\,\exp\left(-\frac{\chi^2(1 - x^2)}{2}\right)$$

with $x = 2M_{bc}/M(\Upsilon)$

Semi-justification for this functional form in next page….

Note how the $D\pi$ background can be distinguished from the DK signal in the ΔE distribution but not in the M_{BC} distribution.

Argus function

- Recall $\mathrm{M_{bc}^2 = E_{expected}^2 - P_{measured}^2\,(B) = \frac{M^2(\Upsilon)}{4} - P^2}$
- <u>Argus function represents random combination (combinatorics) of particles whose measured momentum is P</u>
- $P = \sqrt{\frac{M^2(\Upsilon)}{4} - M_{BC}^2} = \frac{M(\Upsilon)}{2}\sqrt{1 - x^2}$ where as on the previous page $x = \frac{2M_{bc}}{M(\Upsilon)}$
- $dP = \frac{M(\Upsilon)}{2}\frac{(-x)}{\sqrt{1-x^2}}\,dx$
- The volume element in 3d momentum space is

$$dV = P^2 dP = \frac{M^2(\Upsilon)}{4}\left(1 - x^2\right)\frac{M(\Upsilon)}{2}\frac{(-x)\,dx}{\sqrt{1-x^2}} = -\frac{M^3(\Upsilon)}{8}\,x\sqrt{1-x^2}\,dx \propto x\sqrt{1-x^2}\,dx$$

- Phase space is proportional to ***dV*** → expect a function that represents combinatorics to look something like

$$f(x) \sim g(x)\, x\sqrt{1 - x^2}\, dx$$

where *g(x)* is some (~ slowly) function of *x* near *x=1*

- It is found that $g(x) = e^{-\frac{\lambda^2}{2}(1-x^2)}$ where λ is a fitted parameter, works quite well