Poisson Limit, Frequentist vs Bayesian

Let N_0 = number of observed events μ_{90} the 90% limit.
The Poisson distribution of mean μ is

$$p(N|\mu) \;=\; \frac{\mu^N e^{-\mu}}{N!}$$

The frequentist limit is given by solving for μ_{90} such that

$$\begin{aligned} p(N \leq N_0|\mu_{90}) \;&=\; 0.1 \\ \sum_{i=0}^{i=N_0} p(i|\mu_{90}) \;&=\; 0.1 \\ e^{-\mu_{90}} \sum_{i=0}^{i=N_0} \frac{\mu^i}{i!} \;&=\; 0.1 \qquad (1) \end{aligned}$$

The posterior distribution for a Poisson with uniform prior is

$$p(\mu|N)d\mu = \frac{\mu^N e^{-\mu}}{N!} d\mu$$

Note that this is properly normalized

$$\int_0^\infty p(\mu|N)\; d\mu \;=\; \frac{1}{N!}\int_0^\infty \mu^N e^{-\mu}\; d\mu \;=\; \frac{1}{N!}\;\Gamma(N+1,0) \;=\; \frac{\Gamma(N+1)}{N!} \;=\; 1$$

where $\Gamma(a,b)$ is the upper incomplete gamma function. The 90% limit is then given by solving

$$\begin{aligned} \int_{\mu_{90}}^\infty p(\mu|N_0)d\mu \;&=\; 0.1 \\ \frac{1}{N_0!}\int_{\mu_{90}}^\infty \mu^{N_0} e^{-\mu} d\mu \;&=\; 0.1 \\ \frac{1}{N_0!}\;\Gamma(N_0+1,\mu_{90}) \;&=\; 0.1 \\ \frac{1}{N_0!}\;N_0!\;e^{-\mu_{90}} \sum_{i=0}^{i=N_0} \frac{\mu^i}{i!} \;&=\; 0.1 \qquad (2) \end{aligned}$$

where I have used the expansion of $\Gamma(a,b)$ when a is a positive integer. The factorials cancel, and equations (1) and (2) are identical.