# Jeffrey's priors

A good explanation of Jeffrey's priors can be found in this series of three videos

1. `https://tinyurl.com/4scpz2xk`
2. `https://tinyurl.com/wph2z`
3. `https://tinyurl.com/nupd5y3t`

The Jeffrey's prior for one variable is $p_J(\mu)ø \propto \sqrt{I}$, where $I$ is the Fisher information. Under some reasonable assumptions the Fisher information is the negative of the expectation value of the second derivative of the natural log of the likelihood. If you think of the usual negative log-likelihood plot, this second derivative is the curvature at the minimum. The larger the curvature, the "easiest" it is to find the minimum, ie, the higher the information. (OK, this is a lot of handwaving...).
For a Poisson,

$$
\begin{aligned}
p(N|\mu) &= \frac{\mu^N e^{-\mu}}{N!}. \\
\log(p) &= N \log \mu - \mu - \log N! \\
\frac{d^2 log(p)}{d^2\mu} &= \frac{-N}{\mu^2}
\end{aligned}
$$

Since the expectation value $E(N) = \mu$ we have

$$
I = -E\left(\frac{d^2 log(p)}{d^2\mu}\right) = \frac{1}{\mu}
$$

and Jeffrey's prior is

$$
p_J(\mu) \propto \sqrt{I} = \frac{1}{\sqrt{\mu}}
$$

For a Gaussian

$$
\begin{aligned}
p(x|\mu,\sigma) &= \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{\sigma^2}\right) \\
\log(p) &= -\frac{(x-\mu)^2}{\sigma^2} - \log\frac{1}{\sqrt{2\pi}\sigma} \\
\frac{d^2 log(p)}{d^2\mu} &= \frac{-2}{\sigma^2}
\end{aligned}
$$

Therefore the information is

$$I \; = \; -E\left(\frac{d^2 log(p)}{d^2\mu}\right) = \frac{2}{\sigma^2}$$

$\sigma$ is a constant, and the information does not depend on $\mu$, therefore the Jeffrey's prior is a flat uniform distribution:

$$p_J(\mu) = \text{Constant}$$