Srednicki Chapter 69

QFT Problems & Solutions

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Srednicki 69.1. Show that equation 69.9 implies a transformation rule for A^a_μ that is independent of the representation used in equation 69.9. Hint: consider an infinitesimal transformation.

Equation 69.9 is:

$$A_\mu(x) \to U(x)A_\mu(x)U^\dagger(x) + \frac{i}{g}U(x)\partial_\mu U^\dagger(x)$$

Now we need to decompose A into the representation basis vectors with:

$$A_\mu \to A^b_\mu T^b_R$$

The Unitary transformation for an infinitesimal transformation is given by equation 69.6, which we generalize to all representations (and drop the index notation):

$$U(x) = I - ig\theta^a(x)T^a_R$$

Combining all these and dropping the dependences, our transformation becomes:

$$A^b_\mu T^b_R \to [I - ig\theta^a T^a_R]\, A^b_\mu T^b_R\, [I + ig\theta^c T^c_R] + \frac{i}{g}[I - ig\theta^a T^a_R]\left[-ig\left(\partial^\mu\theta^a\right)T^a_R\right]$$

which is, dropping the terms with more than one infinitesimal:

$$A^b_\mu T^b_R \to A^b_\mu T^b_R + igA^b_\mu\left(T^b_R T^c_R - T^a_R T^b_R\right) + \partial^\mu\theta^a T^a_R$$

Since the T_Rs are orthogonal, there is no harm in setting $c = a$. Then:

$$A^b_\mu T^b_R \to A^b_\mu T^b_R - igA^b_\mu\theta^a[T^a_R, T^b_R] + \partial^\mu\theta^a T^a_R$$

Now we use 69.7 to evaluate the commutator:

$$A^b_\mu T^b_R \to A^b_\mu T^b_R - igA^b_\mu\theta^a if^{abc}T^c + \partial^\mu\theta^a T^a_R$$

Now we're making progress, but to be representation-independent, we need to get rid of the T_Rs. Let's factor, relabeling as necessary:

$$A^b_\mu T^b_R \to \left[A^b_\mu + gA^c_\mu f^{acb}\theta^a + \partial^\mu\theta^b\right]T^b_R$$

Now let's multiply on the right by T_R^d:

$$A_\mu^b T_R^b T_R^d \to \left[A_\mu^b + gA_\mu^c f^{acb}\theta^a + \partial^\mu \theta^b\right] T_R^b T_R^d$$

We take the trace of both sides:

$$A_\mu^b \mathrm{Tr}(T_R^b T_R^d) \to \left[A_\mu^b + gA_\mu^c f^{acb}\theta^a + \partial^\mu \theta^b\right] \mathrm{Tr}(T_R^b T_R^d)$$

Now we choose the generators according to equation 69.8:

$$A_\mu^b \to A_\mu^b + gA_\mu^c f^{acb}\theta^a + \partial^\mu \theta^b$$

which is representation-independent.

Srednicki 69.2. Show that $[\boldsymbol{T^a T^a}, \boldsymbol{T^b}] = \mathbf{0}$.

Using the usual properties of the commutators:

$$[T^a T^a, T^b] = T^a[T^a, T^b] + [T^a, T^b]T^a$$

Using equation 69.7:

$$[T^a T^a, T^b] = T^a i f^{abc} T^c + i f^{abc} T^c T^a$$

Now in this second term, we relabel $a \leftrightarrow c$:

$$[T^a T^a, T^b] = T^a i f^{abc} T^c + i f^{cba} T^a T^c$$

Now we use the anti-symmetry of f to obtain:

$$[T^a T^a, T^b] = T^a i f^{abc} T^c - i f^{abc} T^a T^c$$

which is of course:

$$[T^a T^a, T^b] = 0$$

as expected.