Srednicki Chapter 67

QFT Problems & Solutions

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Srednicki 67.1. Show explicitly that the tree level $\tilde{e}^+\tilde{e}^- \to \gamma\gamma$ scattering amplitude in scalar electrodynamics,

$$\boldsymbol{\mathcal{T} = -e^2 \left[\frac{4(k_1 \cdot \varepsilon_{1'})(k_2 \cdot \varepsilon_{2'})}{m^2 - t} + \frac{4(k_1 \cdot \varepsilon_{2'})(k_2 \cdot \varepsilon_{1'})}{m^2 - u} + 2(\varepsilon_{1'} \cdot \varepsilon_{2'}) \right]}$$

vanishes if $\varepsilon^\mu_{1'}$ is replaced with $k^\mu_{1'}$.

We have:

$$\mathcal{T} = -e^2 \left[\frac{4(k_1 \cdot k_{1'})(k_2 \cdot \varepsilon_{2'})}{m^2 - t} + \frac{4(k_1 \cdot \varepsilon_{2'})(k_2 \cdot k_{1'})}{m^2 - u} + 2(k_{1'} \cdot \varepsilon_{2'}) \right]$$

Using our usual definitions of t and u, we have:

$$\mathcal{T} = -e^2 \left[2(k_2 \cdot \varepsilon_{2'}) + 2(k_1 \cdot \varepsilon_{2'}) + 2(k_{1'} \cdot \varepsilon_{2'}) \right]$$

Which is:

$$\mathcal{T} = -2e^2 \left[k_2 + k_1 + k_{1'} \right] \cdot \varepsilon_{2'}$$

This implies:

$$\mathcal{T} = 2e^2 k_{2'} \cdot \varepsilon_{2'}$$

Now polarization is always orthogonal to momentum, so:

$$\mathcal{T} = 0$$

as expected.

Srednicki 67.2. Show explicitely that the tree-level $e^+e^- \to \gamma\gamma$ scattering amplitude in spinor electrodynamics,

$$\boldsymbol{\mathcal{T} = e^2\, \overline{v}_2 \left[\not{\varepsilon}_{2'} \left(\frac{-\not{p}_1 + \not{k}_{1'} + m}{m^2 - t} \right) \not{\varepsilon}_{1'} + \not{\varepsilon}_{1'} \left(\frac{-\not{p}_1 + \not{k}_{2'} + m}{m^2 - u} \right) \not{\varepsilon}_{2'} \right] u_1}$$

vanishes if $\varepsilon^\mu_{1'}$ is replaced with $k^\mu_{1'}$.

We have:

$$\mathcal{T} = e^2\, \overline{v}_2 \left[\not{\varepsilon}_{2'} \left(\frac{-\not{p}_1 + \not{k}_{1'} + m}{m^2 - t} \right) \not{k}_{1'} + \not{k}_{1'} \left(\frac{-\not{p}_1 + \not{k}_{2'} + m}{m^2 - u} \right) \not{\varepsilon}_{2'} \right] u_1$$

We use 37.26:

$$\mathcal{T} = e^2\ \overline{v}_2 \left[\not{e}_{2'} \left(\frac{-\not{p}_1 + m}{m^2 - t} \right) \not{k}_{1'} + \not{k}_{1'} \left(\frac{-\not{p}_1 + \not{k}_{2'} + m}{m^2 - u} \right) \not{e}_{2'} \right] u_1$$

From conservation of momentum:

$$\mathcal{T} = e^2\ \overline{v}_2 \left[\not{e}_{2'} \left(\frac{-\not{p}_1 + m}{m^2 - t} \right) \not{k}_{1'} + \not{k}_{1'} \left(\frac{\not{p}_2 - \not{k}_{1'} + m}{m^2 - u} \right) \not{e}_{2'} \right] u_1$$

Using 37.26 again:

$$\mathcal{T} = e^2\ \overline{v}_2 \left[\not{e}_{2'} \left(\frac{-\not{p}_1 + m}{m^2 - t} \right) \not{k}_{1'} + \not{k}_{1'} \left(\frac{\not{p}_2 + m}{m^2 - u} \right) \not{e}_{2'} \right] u_1$$

Rewriting this:

$$\mathcal{T} = e^2\ \overline{v}_2 \left[\frac{-\not{e}_{2'}\not{p}_1\not{k}_{1'} + \not{e}_{2'}\not{k}_{1'}m}{m^2 - t} + \frac{\not{k}_{1'}\not{p}_2\not{e}_{2'} + \not{k}_{1'}\not{e}_{2'}m}{m^2 - u} \right] u_1$$

Now we use 47.10:

$$\mathcal{T} = e^2\ \overline{v}_2 \left[\frac{\not{e}_{2'}\left(\not{k}_{1'}\not{p}_1 + 2(p_1 \cdot k_{1'})\right) + \not{e}_{2'}\not{k}_{1'}m}{m^2 - t} + \frac{-\left(\not{p}_2\not{k}_{1'} + 2(p_2 \cdot k_{1'})\right)\not{e}_{2'} + \not{k}_{1'}\not{e}_{2'}m}{m^2 - u} \right] u_1$$

Now we use 37.29 (the Dirac Equation); this causes the first and the third terms in each numerator to cancel:

$$\mathcal{T} = 2e^2\ \overline{v}_2 \left[\frac{\not{e}_{2'}\,(p_1 \cdot k_{1'})}{m^2 - t} - \frac{(p_2 \cdot k_{1'})\,\not{e}_{2'}}{m^2 - u} \right] u_1$$

Rewriting this:

$$\mathcal{T} = 2e^2\ \overline{v}_2\not{e}_{2'} \left[\frac{(p_1 \cdot k_{1'})}{m^2 - t} - \frac{(p_2 \cdot k_{1'})}{m^2 - u} \right] u_1$$

Now we have $t = -(p_1 - k_{1'})^2 = m^2 + 2p_1 \cdot k_{1'}$ and $u = -(p_2 - k_{1'})^2 = m^2 + 2p_2 \cdot k_{1'}$. Then:

$$\mathcal{T} = 2e^2\ \overline{v}_2\not{e}_{2'} \left[-\frac{1}{2} + \frac{1}{2} \right] u_1$$

and so:

$$\mathcal{T} = 0$$

as expected.