Srednicki Chapter 65

QFT Problems & Solutions

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Srednicki 65.1. What conditions should be imposed on $V_3^\mu(p', p)$ and $V_4^{\mu\nu}(k, p', p)$ in the OS scheme? (Here k is the incoming four-momentum of the photon at the μ vertex, and p' and p are the four-momenta of the outgoing and incoming scalars, respectively.)

The OS condition involves setting the m parameters equal to the physical masses, and choosing the Z factors to cancel everything up to a numeric term. By convention, this numeric term is set to zero, so the exact vertex function is equal to the tree level value. In ϕ^3 theory the tree-level function is g.

Using part of equation 65.3, the first vertex is:

$$\text{Vertex} = iZ_1 e[\phi^\dagger \partial^\mu \phi - (\partial^\mu \phi^\dagger)\phi]A_\mu$$

First we replace the partials with ik:

$$\text{Vertex} = iZ_1 e[\phi^\dagger i p_1^\mu \phi - (i p_2^\mu \phi^\dagger)\phi]A_\mu$$

Next we scratch out the fields and add a factor of i:

$$\text{Vertex} = i^2 Z_1 e[i p_1^\mu - i p_2^\mu]$$

which is:

$$\text{Vertex} = -iZ_1 e[p_1^\mu - p_2^\mu]$$

Now p_2 is outgoing, but these diagrams by convention are drawn as incoming. Thus:

$$\text{Vertex} = -iZ_1 e[p_1^\mu + p_2^\mu]$$

Now this is the result from the tree-level diagram, which is equal to iV_3:

$$iV_3(p', p) = -iZ_1 e[p^\mu + p'^\mu]$$

This gives:

$$V_3(p', p) = -Z_1 e[p^\mu + p'^\mu]$$

As discussed, the Z_1 is chosen to cancel the higher terms, so our answer is:

$$\boxed{V_3(p', p) = -e[p^\mu + p'^\mu]}$$

For the two photon, two scalar vertex, we again scratch out the fields, add in a factor of i, multiply through by the symmetry factor (two, from swapping the two scalars), and equate this to iV_4. Note that the fields are defined as "index up" (or index down), so we have to add in g-factors before we can strike out the fields. The result is:

$$\boxed{V_4^{\mu\nu}(p', p, k) = -2e^2 g^{\mu\nu}}$$

Srednicki 65.2. Consider a gauge transformation $A^\mu \to A^\mu - \partial^\mu \Gamma$. Show that there is a transformation of ϕ that leaves the Lagrangian of equations 65.1-65.4 invariant if and only if $Z_4 = Z_1^2/Z_2$.

The obvious way to solve this is to take the transformation on A ($A^\mu \to A^\mu - \partial^\mu\Gamma$), to apply the most general transformation on ϕ ($\phi \to e^{i\alpha}\phi$), and to insist that there be no change to the Langranian. But, this will be a mess.

Instead, we follow Srednicki's unintuitive approach. We define the a generic covariant derivative by:

$$D^\mu = \partial^\mu - iKeA^\mu$$

Now we have from equation 58.9:

$$D^\mu\phi \to e^{-ie\Gamma} D^\mu\phi$$

for $A^\mu \to A^\mu - \partial^\mu\Gamma$ and $\phi \to e^{-ieK\Gamma}\phi$.

Now $D^\mu\phi^\dagger D_\mu\phi$ is manifestly invariant under these two transformations. This means:

$$\text{invt} = (\partial^\mu - iKeA^\mu)\phi^\dagger(\partial_\mu - iKeA_\mu)\phi$$

which is:

$$\text{invt} = \partial^\mu\phi^\dagger\partial_\mu\phi - iKeA^\mu\phi^\dagger\partial_\mu\phi + \partial^\mu\phi^\dagger(-iKe)A_\mu\phi - K^2e^2A^\mu\phi^\dagger A_\mu\phi$$

Now we multiply everything by $-Z_2$:

$$\text{invt} = -Z_2\partial^\mu\phi^\dagger\partial_\mu\phi + iKeZ_2A^\mu\phi^\dagger\partial_\mu\phi - Z_2\partial^\mu\phi^\dagger(-iKe)A_\mu\phi + Z_2K^2e^2A^\mu\phi^\dagger A_\mu\phi$$

Now we can set $Z_2 = Z_1/K$ in the second and third terms, and $Z_2 = Z_4/k^2$ in the third term, to reproduce those four terms of the Lagrangian specified. Solving these conditions gives $Z_4 = Z_1^2/Z_2$ as specified.

Now we're done up to the remaining terms, which are $\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ (invariant as per 58.14) and $Z_m m^2\phi^\dagger\phi$ (manifestly invariant under the transformations specified above). Thus, these terms can be added to the Lagrangian while leaving it invariant.

Note: Actually, this is perhaps not so unintuitive as I claimed. The Lagrangian must be invariant, which means it must be able to be written in terms of invariant things, like covariant derivatives. This does not mean that we can just arbitrarily replace our partial derivatives with covariant ones! Rather it means that one could factor the partial derivatives into covariant derivatives. Here, we simply approach the problem from the other direction.