Srednicki Chapter 63

QFT Problems & Solutions

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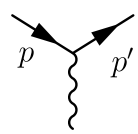
Srednicki 63.1. The most general possible form of $\overline{u}'V^\mu(p',p)u$ is a linear combination of γ^μ, p^μ, $(p')^\mu$ sandwiched between $\overline{u}'$ and u, with coefficients that depend on q^2. (The only other possibility is to include terms with γ_5, but γ_5 does not appear in the tree-level propagators or vertex, and so it cannot be generated in any Feynman diagram; this is a consequence of parity conservation.) Thus we can write

$$\overline{u}_{s'}(p')V^\mu(p',p)u_s(p) = e\overline{u}'\left[A(q^2)\gamma^\mu + B(q^2)(p'+p)^\mu + C(q^2)(p'-p)^\mu\right]u$$

(a) Use gauge invariance to show that $q_\mu\overline{u}'V^\mu(p',p)u = 0$, and determine the consequences for A, B, C.

Consider the following diagram:



Let's say that the photon line leads to a source, so we will treat the photon as an internal photon. Then we assess the value of this as:

$$i\mathcal{T} = \overline{u}(p')V^\mu(p,p')u(p)\frac{-i(g_{\mu\nu}+\alpha q_\mu q_\nu)}{q^2 - i\varepsilon}$$

where q is the photon momentum, and we use the general term in Feynman Gauge (the result in chapter 56 dropped the second term; we're going to keep it for the moment. The coefficient doesn't matter, so we'll just call it α). Now this observable should be gauge-independent. As discussed in chapter 56, these terms with q are irrelevant in the Feynman gauge, because as discussed, they can be written in terms of conserved currents. If they are irrelevant in the Feynman gauge, they should be irrelevant in all gauges. Thus, the $q_\mu q_\nu$ terms must therefore vanish; the only way to require this is to have

$$q_\mu\overline{u}V^\mu u = 0$$

Given this, we note that the photon momentum q is $p - p'$. So, we have:

$$\overline{u}'(p'-p)_\mu\left[A\gamma^\mu + B(p'+p)^\mu + C(p'-p)^\mu\right]u = 0$$

Distributing the q, the first term vanishes by the Dirac Equation, and the second equation vanishes because $p^2 = (p')^2 = -m^2$. Then, we have:

$$C(p' - p)^2 = 0$$

which is only possible if C = 0. That's all we can say; with only one constraint and three free parameters, the other two parameters are still free.

(b) Express F_1 and F_2 in terms of A, B, C.

Equation 63.23 gives, after our result from part (a):

$$\overline{u}'Vu = e\overline{u}'\left[A\gamma^\mu + B(p' + p)\right]u$$

Now use equation 63.16, the Gordon Identity:

$$\overline{u}'Vu = e\overline{u}'\left[A\gamma^\mu + B2m\gamma^\mu + B2iS^{\mu\nu}q_\nu\right]u$$

Reorganizing this:

$$\overline{u}'Vu = e\overline{u}'\left[(A + 2mB)\gamma^\mu - \frac{i}{m}\left(-2mB\right)S^{\mu\nu}q_\nu\right]u$$

Comparing this to equation 63.18, we compute the form factors: $F_1(q^2) = A + 2mB$, $F_2(q^2) = -2mB$.