Srednicki Chapter 61

QFT Problems & Solutions

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Srednicki 61.1. Compute $\langle|\mathcal{T}|^2\rangle$ for $\tilde{e}^+\tilde{e}^- \to \gamma\gamma$, and express your answer in terms of the Mandelstam Variables.

Mercifully, Srednicki started this one for us. We pick up where he left off, with equation (61.14):

$$\mathcal{T} = -e^2\left[\frac{4(k_1\cdot\varepsilon_{1'})(k_2\cdot\varepsilon_{2'})}{m^2-t} + \frac{4(k_1\cdot\varepsilon_{2'})(k_2\cdot\varepsilon_{1'})}{m^2-u} + 2(\varepsilon_{1'}\cdot\varepsilon_{2'})\right]$$

We can factor this a little bit:

$$\mathcal{T} = -e^2\varepsilon_{1'\mu}\varepsilon_{2'\nu}\left[\frac{4k_1^\mu k_2^\nu}{m^2-t} + \frac{4k_1^\nu k_2^\mu}{m^2-u} + 2g^{\mu\nu}\right]$$

We can take the conjugate:

$$\overline{\mathcal{T}} = -e^2\varepsilon^*_{1'\rho}\varepsilon^*_{2'\sigma}\left[\frac{4k_1^\rho k_2^\sigma}{m^2-t} + \frac{4k_1^\sigma k_2^\rho}{m^2-u} + 2g^{\rho\sigma}\right]$$

This gives:

$$|\mathcal{T}|^2 = 4e^4\varepsilon_{1'\mu}\varepsilon_{2'\nu}\varepsilon^*_{1'\rho}\varepsilon^*_{2'\sigma}\left[\frac{2k_1^\mu k_2^\nu}{m^2-t} + \frac{2k_1^\nu k_2^\mu}{m^2-u} + g^{\mu\nu}\right]\left[\frac{2k_1^\rho k_2^\sigma}{m^2-t} + \frac{2k_1^\sigma k_2^\rho}{m^2-u} + g^{\rho\sigma}\right]$$

As Srednicki indicated, we use (61.15) to sum over the final states. Recall that the selectrons are scalars, which have no spins to sum over:

$$\langle|\mathcal{T}|^2\rangle = 4e^4 g_{\mu\rho}g_{\nu\sigma}\left[\frac{2k_1^\mu k_2^\nu}{m^2-t} + \frac{2k_1^\nu k_2^\mu}{m^2-u} + g^{\mu\nu}\right]\left[\frac{2k_1^\rho k_2^\sigma}{m^2-t} + \frac{2k_1^\sigma k_2^\rho}{m^2-u} + g^{\rho\sigma}\right]$$

Using the metric:

$$\langle|\mathcal{T}|^2\rangle = 4e^4\left[\frac{2k_1^\mu k_2^\nu}{m^2-t} + \frac{2k_1^\nu k_2^\mu}{m^2-u} + g^{\mu\nu}\right]\left[\frac{2k_{1\mu}k_{2\nu}}{m^2-t} + \frac{2k_{1\nu}k_{2\mu}}{m^2-u} + g_{\mu\nu}\right] \qquad (61.1.1)$$

Next we distribute:

$$\langle|\mathcal{T}|^2\rangle = 4e^4\left[\frac{4(k_1\cdot k_1)(k_2\cdot k_2)}{(m^2-t)^2} + \frac{8(k_1\cdot k_2)(k_1\cdot k_2)}{(m^2-t)(m^2-u)} + \frac{4(k_1\cdot k_2)}{m^2-t} + \frac{4(k_1\cdot k_1)(k_2\cdot k_2)}{(m^2-u)^2}\right.$$

$$+\frac{4(k_1\cdot k_2)}{m^2-u}+4\Bigg]$$

This gives:

$$\langle|\mathcal{T}|^2\rangle = 16e^4\left[\frac{(k_1\cdot k_1)(k_2\cdot k_2)}{(m^2-t)^2}+\frac{2(k_1\cdot k_2)(k_1\cdot k_2)}{(m^2-t)(m^2-u)}+\frac{(k_1\cdot k_2)}{2(m^2-t)}+\frac{(k_1\cdot k_1)(k_2\cdot k_2)}{(m^2-u)^2}\right.$$
$$\left.+\frac{(k_1\cdot k_2)}{m^2-u}+1\right]$$

Next we need to put in Mandelstam variables:

$$k_1^2 = k_2^2 = -m^2$$

$$s = -(k_1+k_2)^2 = 2m^2 - 2k_1\cdot k_2 \implies k_1\cdot k_2 = \frac{2m^2-s}{2}$$

This gives:

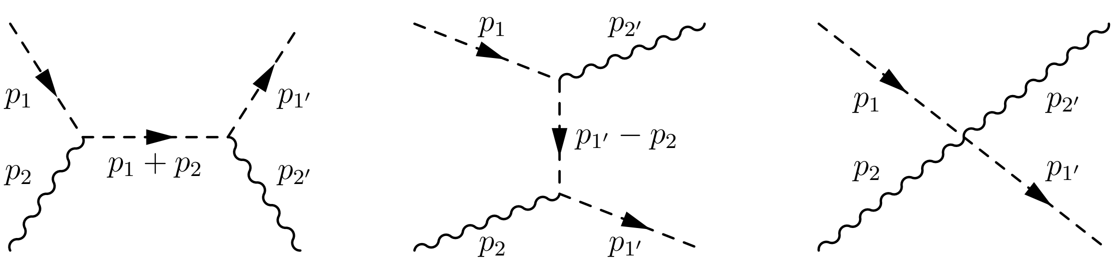
$$\langle|\mathcal{T}|^2\rangle = 16e^4\left[\frac{m^4}{(m^2-t)^2}+\frac{(s-2m^2)^2}{2(m^2-t)(m^2-u)}+\frac{2m^2-s}{2(m^2-t)}+\frac{m^4}{(m^2-u)^2}\right.$$
$$\left.+\frac{2m^2-s}{2(m^2-u)}+1\right]$$

By the way, $2m^2-s = t+u$:

$$\boxed{\langle|\mathcal{T}|^2\rangle = 16e^4\left[\frac{m^4}{(m^2-t)^2}+\frac{(t+u)^2}{2(m^2-t)(m^2-u)}+\frac{t+u}{2(m^2-t)}+\frac{m^4}{(m^2-u)^2}+\frac{t+u}{2(m^2-u)}+1\right]}$$

Srednicki 61.2. Compute $\langle|\mathcal{T}|^2\rangle$ for the process $\tilde{e}^-\gamma \to \tilde{e}^-\gamma$. You should find that your result is the same as that for $\tilde{e}^+\tilde{e}^- \to \gamma\gamma$, but with $s \leftrightarrow t$, an example of crossing symmetry.

We begin with the diagrams:



Note that the arrows are simply momentum arrows; these are scalars, not fermions. We further neglect the arrows on the photons, as their directions are obvious (forward direction). We use these to assess the amplitude:

$$i\mathcal{T} = \varepsilon^{\mu *}_{\lambda 2}(p_2)\varepsilon^{\nu}_{\lambda 2'}(p_{2'})ie(2p_1+p_2)_\mu ie(p_1+p_2+p_{2'})\frac{-i}{(p_1+p_2)^2+m^2-i\varepsilon}$$

$$+\varepsilon^{\mu *}_{\lambda 2}(p_2)\varepsilon^{\nu}_{\lambda 2'}(p_{2'})ie(2p_1 - p_{2'})_\nu ie(p_1 + p_2 - p_{2'})_\mu \frac{-i}{(p_1 - p_{2'})^2 + m^2 - i\varepsilon}$$

$$+\varepsilon^{\mu *}_{\lambda 2}(p_2)\varepsilon^{\nu}_{\lambda 2'}(p_{2'})(-2)ie^2 g_{\mu\nu}$$

Simplifying, and using the Mandelstam variables in the denominator:

$$\mathcal{T} = e^2\varepsilon^{\mu *}_{\lambda 2}(p_2)\varepsilon^{\nu}_{\lambda 2'}(p_{2'})\left[\frac{(2p_1 + p_2)_\mu(p_1 + p_2 + p_{1'})_\nu}{-s + m^2} + \frac{(2p_1 - p_{2'})_\nu(p_1 - p_{2'} + p_{1'})_\mu}{-u + m^2} - 2g_{\mu\nu}\right]$$

Next we recall that $p_1 + p_2 = p_{1'} + p_{2'}$, and that $p_i \cdot \varepsilon_{\lambda i}(p_i) = 0$. Using this, we can simplify these terms like this:

$$(2p_1 - p_{2'}) \cdot \varepsilon_{2'}(p_{2'}) = 2p_1 \cdot \varepsilon_{2'}$$

$$(2p_1 + p_2) \cdot \varepsilon_2(p_2) = 2p_1 \cdot \varepsilon_2$$

$$(p_1 + p_{1'} - p_{2'}) \cdot \varepsilon_2(p_2) = (2p_{1'} - p_2) \cdot \varepsilon_2 = 2p_{1'} \cdot \varepsilon_2$$

$$(p_1 + p_2 + p_{1'}) \cdot \varepsilon_{2'} = (2p_{1'} + p_{2'}) \cdot \varepsilon_{2'} = 2p_{1'} \cdot \varepsilon_{2'}$$

Using these:

$$\mathcal{T} = 2e^2\varepsilon^{\mu *}_{\lambda 2}(p_2)\varepsilon^{\nu}_{2'}(p_{2'})\left[\frac{2p_{1\mu}p_{1'\nu}}{-s + m^2} + \frac{2p_{1\nu}p_{1'\mu}}{-u + m^2} - g_{\mu\nu}\right]$$

Taking the conjugate:

$$\overline{\mathcal{T}} = 2e^2\varepsilon^{\sigma}_{\lambda 2}(p_2)\varepsilon^{\rho *}_{2'}(p_{2'})\left[\frac{2p_{1\sigma}p_{1'\rho}}{-s + m^2} + \frac{2p_{1\rho}p_{1'\sigma}}{-u + m^2} - g_{\sigma\rho}\right]$$

This gives:

$$|\mathcal{T}|^2 = 2e^4\varepsilon^{\mu *}_{\lambda 2}(p_2)\varepsilon^{\nu}_{2'}(p_{2'})\varepsilon^{\sigma}_{\lambda 2}(p_2)\varepsilon^{\rho *}_{2'}(p_{2'})\left[\frac{2p_{1\mu}p_{1'\nu}}{-s + m^2} + \frac{2p_{1\nu}p_{1'\mu}}{-u + m^2} - g_{\mu\nu}\right]\left[\frac{2p_{1\sigma}p_{1'\rho}}{-s + m^2} + \frac{2p_{1\rho}p_{1'\sigma}}{-u + m^2} - g_{\sigma\rho}\right]$$

Next we sum over the final states and average over the inital states à la (61.15):

$$\langle|\mathcal{T}|^2\rangle = e^4 g^{\mu\sigma}g^{\nu\rho}\left[\frac{2p_{1\mu}p_{1'\nu}}{-s + m^2} + \frac{2p_{1\nu}p_{1'\mu}}{-u + m^2} - g_{\mu\nu}\right]\left[\frac{2p_{1\sigma}p_{1'\rho}}{-s + m^2} + \frac{2p_{1\rho}p_{1'\sigma}}{-u + m^2} - g_{\sigma\rho}\right]$$

Using the metric:

$$\langle|\mathcal{T}|^2\rangle = e^4\left[\frac{2p_{1\mu}p_{1'\nu}}{-s + m^2} + \frac{2p_{1\nu}p_{1'\mu}}{-u + m^2} - g_{\mu\nu}\right]\left[\frac{2p_1^\mu p_{1'}^\nu}{-s + m^2} + \frac{2p_1^\nu p_{1'}^\mu}{-u + m^2} - g^{\mu\nu}\right]$$

This is the same as equation (61.1.1) with $p_{1'} \leftrightarrow -p_2$ (which corresponds to $t \leftrightarrow s$) and with a factor of $\frac{1}{2}$. Thus, the answer is the same up to these modifications:

$$\boxed{\langle|\mathcal{T}|^2\rangle = 8e^4\left[\frac{m^4}{(m^2 - s)^2} + \frac{(s + u)^2}{2(m^2 - s)(m^2 - u)} + \frac{s + u}{2(m^2 - s)} + \frac{m^4}{(m^2 - u)^2} + \frac{s + u}{2(m^2 - u)} + 1\right]}$$