Srednicki Chapter 60

QFT Problems & Solutions

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Srednicki 60.1. (a) Show that

$$\boldsymbol{p \cdot \varepsilon_+(k;q) = \frac{\langle qp\rangle [pk]}{\sqrt{2}\langle qk\rangle}}$$

$$\boldsymbol{p \cdot \varepsilon_-(k;q) = \frac{[qp]\langle pk\rangle}{\sqrt{2}[qk]}}$$

Use this result to show that

$$\boldsymbol{k \cdot \varepsilon_\pm(k;q) = 0}$$

which is required by gauge invariance, and also that

$$\boldsymbol{q \cdot \varepsilon_\pm(k;q) = 0}$$

We begin by using (60.7) and (60.8):

$$p \cdot \varepsilon_+ = \frac{\langle q|(-\not{p})|k]}{\sqrt{2}\langle qk\rangle}$$

$$p \cdot \varepsilon_- = \frac{[q|(-\not{p})|k\rangle}{\sqrt{2}[qk]}$$

Next we use (60.6):

$$p \cdot \varepsilon_+ = \frac{\langle q|\left(|p\rangle[p| + |p]\langle p|\right)|k]}{\sqrt{2}\langle qk\rangle}$$

$$p \cdot \varepsilon_- = \frac{[q|\left(|p\rangle[p| + |p]\langle p|\right)|k\rangle}{\sqrt{2}[qk]}$$

And now for (60.2):

$$p \cdot \varepsilon_+ = \frac{\langle qp\rangle [pk]}{\sqrt{2}\langle qk\rangle} \tag{60.1.1}$$

$$p \cdot \varepsilon_- = \frac{[qp]\langle pk\rangle}{\sqrt{2}[qk]} \tag{60.1.2}$$

Now we substitute $p \to k$:

$$k \cdot \varepsilon_+ = \frac{\langle qk\rangle[kk]}{\sqrt{2}\langle qk\rangle}$$

$$k \cdot \varepsilon_- = \frac{[qk]\langle kk\rangle}{\sqrt{2}[qk]}$$

Both of these vanish by 60.3. Obviously, repeating this procedure with the substitution $p \to q$ in (60.1.1) and (60.1.2) will lead to terms which vanish for the same reason. This completes our proof.

(b) Show that

$$\boldsymbol{\varepsilon_+(k;q) \cdot \varepsilon_-(k';q') = \frac{\langle qq'\rangle[kk']}{\langle qk\rangle\langle q'k'\rangle}}$$

$$\boldsymbol{\varepsilon_-(k;q) \cdot \varepsilon_-(k';q') = \frac{[qq']\langle kk'\rangle}{[qk][q'k']}}$$

$$\boldsymbol{\varepsilon_+(k;q) \cdot \varepsilon_+(k';q') = \frac{\langle qk'\rangle[kq']}{\langle qk\rangle[q'k']}}$$

We use equations (60.7) and (60.8):

$$\varepsilon_+(k;q) \cdot \varepsilon_-(k';q') = \frac{\langle q|\gamma^\mu|k][q'|\gamma_\mu|k'\rangle}{2\langle qk\rangle[q'k']}$$

$$\varepsilon_-(k;q) \cdot \varepsilon_-(k';q') = \frac{[q|\gamma^\mu|k\rangle[q'|\gamma_\mu|k'\rangle}{2[qk][q'k']}$$

$$\varepsilon_+(k;q) \cdot \varepsilon_+(k';q') = \frac{\langle q|\gamma^\mu|k]\langle q'|\gamma_\mu|k']}{2\langle qk\rangle\langle q'k'\rangle}$$

Now we use (50.38) in the second and third equations:

$$\varepsilon_+(k;q) \cdot \varepsilon_-(k';q') = \frac{[q'|\gamma_\mu|k'\rangle\langle q|\gamma^\mu|k]}{2\langle qk\rangle[q'k']}$$

$$\varepsilon_-(k;q) \cdot \varepsilon_-(k';q') = \frac{[q|\gamma^\mu|k\rangle\langle k'|\gamma_\mu|q']}{2[qk][q'k']}$$

$$\varepsilon_+(k;q) \cdot \varepsilon_+(k';q') = \frac{[k|\gamma^\mu|q\rangle\langle q'|\gamma_\mu|k']}{2\langle qk\rangle\langle q'k'\rangle}$$

We use (50.45):

$$\varepsilon_+(k;q) \cdot \varepsilon_-(k';q') = \frac{[q'k]\langle k'q\rangle}{\langle qk\rangle[q'k']}$$

$$\varepsilon_-(k;q) \cdot \varepsilon_-(k';q') = \frac{[qq']\langle kk'\rangle}{[qk][q'k']}$$

$$\varepsilon_+(k;q) \cdot \varepsilon_+(k';q') = \frac{[kk']\langle qq'\rangle}{\langle qk\rangle\langle q'k'\rangle}$$

Using equation (60.3):

$$\varepsilon_+(k;q)\cdot\varepsilon_-(k';q') = \frac{[kq']\langle qk'\rangle}{\langle qk\rangle[q'k']}$$
$$\varepsilon_-(k;q)\cdot\varepsilon_-(k';q') = \frac{[qq']\langle kk'\rangle}{[qk][q'k']}$$
$$\varepsilon_+(k;q)\cdot\varepsilon_+(k';q') = \frac{[kk']\langle qq'\rangle}{\langle qk\rangle\langle q'k'\rangle}$$

as expected.

Srednicki 60.2. (a) For a process with n external particles, and all momenta treated as outgoing show that:

$$\sum_{j=1}^{n}\langle ij\rangle[jk] = 0 \quad \text{and} \quad \sum_{j=1}^{n}[ij]\langle jk\rangle = 0$$

We note that:

$$p_1 + p_2 + p_3 + \ldots + p_n = 0$$

Slashing this:

$$\not{p}_1 + \not{p}_2 + \not{p}_3 + \ldots + \not{p}_n = 0$$

Using 60.6:

$$\sum_{j=1}^{n}[-|j\rangle[j| - |j]\langle j|] = 0$$

Bracketing and then "angling", and multiplying through by -1:

$$\sum_{j=1}^{n}[ij\rangle[j| + [ij]\langle j| = 0$$

$$\sum_{j=1}^{n}\langle ij\rangle[j| + \langle ij]\langle j| = 0$$

Two of these terms vanish, by (60.2):

$$\sum_{j=1}^{n}[ij]\langle j| = 0$$

$$\sum_{j=1}^{n}\langle ij\rangle[j| = 0$$

Bracketing or "angling" – choosing the one that doesn't vanish – we have:

$$\sum_{j=1}^{n}[ij]\langle jk\rangle = 0$$

$$\sum_{j=1}^{n} \langle ij \rangle [jk] = 0$$

as expected.

(b) For n = 4, show that $[31]\langle 12\rangle = -[34]\langle 42\rangle$

We have, using the result from part (a):

$$[31]\langle 12\rangle = -[32]\langle 22\rangle - [33]\langle 32\rangle - [34]\langle 42\rangle$$

The first two terms vanish by anti-symmetry; the third gives:

$$[31]\langle 12\rangle = -[34]\langle 42\rangle$$

Srednicki 60.3. Use various identities to show that equation 60.31 can also be written as

$$\boldsymbol{\mathcal{T}_{+-+-} = -2e^2 \frac{[13]^2}{[14][24]}}$$

We multiply the denominator and numerator of equation (60.3) by [24][24]:

$$\mathcal{T}_{+-+-} = 2e^2 \frac{\langle 24\rangle\langle 24\rangle[24][24]}{\langle 13\rangle\langle 23\rangle[24][24]}$$

Equation (60.22) gives:

$$\mathcal{T}_{+-+-} = 2e^2 \frac{s_{24}^2}{\langle 13\rangle\langle 23\rangle[24][24]}$$

Stated in the middle "paragraph" of page 367, we have $s_{13} = s_{24}$. Then:

$$\mathcal{T}_{+-+-} = 2e^2 \frac{s_{13}^2}{\langle 13\rangle\langle 23\rangle[24][24]}$$

Use (60.22) again:

$$\mathcal{T}_{+-+-} = 2e^2 \frac{\langle 13\rangle\langle 13\rangle[13][13]}{\langle 13\rangle\langle 23\rangle[24][24]}$$

Cancelling:

$$\mathcal{T}_{+-+-} = 2e^2 \frac{\langle 13\rangle[13][13]}{\langle 23\rangle[24][24]}$$

Using antisymmetry:

$$\mathcal{T}_{+-+-} = -2e^2 \frac{\langle 13\rangle[13][13]}{[42]\langle 23\rangle[24]}$$

Now we use the result of 60.2(b):

$$\mathcal{T}_{+-+-} = 2e^2 \frac{\langle 13\rangle[13][13]}{[41]\langle 13\rangle[24]}$$

Cancelling:

$$\mathcal{T}_{+-+-} = 2e^2 \frac{[13][13]}{[41][24]}$$

Using antisymmetry:

$$\mathcal{T}_{+-+-} = -2e^2 \frac{[13]^2}{[14][24]}$$

as expected.

Srednicki 60.4. (a) Show explicitly that you would get the same result as equation 60.31 if you set $q_4 = p_1$ in equation 60.29.

Equation 60.29 is:

$$\mathcal{T}_{+-+-} = -e^2 \frac{\sqrt{2}}{[q_4 4]} \langle 24 \rangle [q_4|(\not{p}_1 + \not{k}_3)|2\rangle [31] \frac{\sqrt{2}}{\langle 23 \rangle} \frac{1}{s_{13}}$$

Taking $q_4 = p_1$:

$$\mathcal{T}_{+-+-} = -e^2 \frac{\sqrt{2}}{[14]} \langle 24 \rangle [1|(\not{p}_1 + \not{k}_3)|2\rangle [31] \frac{\sqrt{2}}{\langle 23 \rangle} \frac{1}{s_{13}}$$

Using equation 60.6:

$$\mathcal{T}_{+-+-} = \frac{2e^2 \langle 24 \rangle [31]}{[14] \langle 23 \rangle s_{13}} \left([1| \left(|1\rangle[1| + |1]\langle 1| \right) |2\rangle + [1| \left(|3\rangle[3| + |3]\langle 3| \right) |2\rangle \right)$$

This gives:

$$\mathcal{T}_{+-+-} = \frac{2e^2 \langle 24 \rangle [31][13] \langle 32 \rangle}{[14] \langle 23 \rangle [13] \langle 31 \rangle}$$

Using the result from equation 60.2(b):

$$\mathcal{T}_{+-+-} = -\frac{2e^2 \langle 24 \rangle [31][14] \langle 42 \rangle}{[14] \langle 23 \rangle [13] \langle 31 \rangle}$$

Using antisymmetry, and cancelling common factors:

$$\mathcal{T}_{+-+-} = \frac{2e^2 \langle 24 \rangle^2}{\langle 13 \rangle \langle 23 \rangle}$$

as expected.

(b) Show explicitly that you would get the same result as equation 60.31 if you set $q_4 = p_1$ in equation 60.29.

Equation 60.29 is:

$$\mathcal{T}_{+-+-} = -e^2 \frac{\sqrt{2}}{[q_4 4]} \langle 24 \rangle [q_4|(\not{p}_1 + \not{k}_3)|2\rangle [31] \frac{\sqrt{2}}{\langle 23 \rangle} \frac{1}{s_{13}}$$

Taking $q_4 = p_2$:

$$\mathcal{T}_{+-+-} = -e^2 \frac{\sqrt{2}}{[24]} \langle 24 \rangle [2|(\not{p}_1 + \not{k}_3)|2\rangle [31] \frac{\sqrt{2}}{\langle 23 \rangle} \frac{1}{s_{13}}$$

This is:

$$\mathcal{T}_{+-+-} = -2e^2 \frac{\langle 24 \rangle \left([2|\not{p}_1|2\rangle + [2|\not{k}_3|2\rangle \right) [31]}{[24]\langle 23 \rangle \langle 13 \rangle [31]}$$

Using equation 60.6:

$$\mathcal{T}_{+-+-} = 2e^2 \frac{\langle 24 \rangle \left([21]\langle 12 \rangle + [23]\langle 32 \rangle \right) [31]}{[24]\langle 23 \rangle \langle 13 \rangle [31]}$$

This is:

$$\mathcal{T}_{+-+-} = 2e^2 \frac{\langle 24 \rangle (s_{12} + s_{23}) [31]}{[24]\langle 23 \rangle \langle 13 \rangle [31]}$$

Since the Mandelstam variables have a zero sum, and are symmetric ($s_{12} = \langle 12 \rangle [21] = (-1)^2 \langle 21 \rangle [12] = s_{21}$), we have:

$$\mathcal{T}_{+-+-} = -2e^2 \frac{\langle 24 \rangle (s_{24}) [31]}{[24]\langle 23 \rangle \langle 13 \rangle [31]}$$

This gives:

$$\mathcal{T}_{+-+-} = -2e^2 \frac{\langle 24 \rangle [24] \langle 42 \rangle [31]}{[24]\langle 23 \rangle \langle 13 \rangle [31]}$$

Using antisymmetry and cancelling like terms:

$$\mathcal{T}_{+-+-} = 2e^2 \frac{\langle 24 \rangle \langle 24 \rangle}{\langle 23 \rangle \langle 13 \rangle}$$

as expected.