Srednicki Chapter 58

QFT Problems & Solutions

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Srednicki 58.1. Compute $P^{-1}A^{\mu}(x,t)P$, $T^{-1}A^{\mu}(x,t)T$, $C^{-1}A^{\mu}(x,t)C$, assuming that P, T, and C are symmetries of the lagrangian.

We consider the last term of the Lagrangian, $\mathcal{L} = e\overline{\Psi}\gamma^{\mu}\Psi A_{\mu}$. This is the only term with an e, and so it must respect all three symmetries by itself; the other terms cannot contribute.

We begin with P symmetry:

$$P^{-1}e\overline{\Psi}\gamma^{\mu}\Psi A_{\mu}P = e\overline{\Psi}\gamma^{\mu}\Psi A_{\mu}$$

Inserting an identity:

$$eP^{-1}\overline{\Psi}\gamma^{\mu}\Psi PP^{-1}A_{\mu}P = e\overline{\Psi}\gamma^{\mu}\Psi A_{\mu}$$

Raising the index:

$$eP^{-1}\overline{\Psi}\gamma^{\mu}\Psi PP^{-1}A^{\alpha}Pg_{\alpha\mu} = e\overline{\Psi}\gamma^{\alpha}\Psi A_{\mu}$$

Using 40.37:

$$e\mathcal{P}^{\mu}_{\ \nu}\overline{\Psi}\gamma^{\nu}\Psi P^{-1}A^{\alpha}Pg_{\alpha\mu} = e\overline{\Psi}\gamma^{\mu}\Psi A_{\mu}$$

To enforce this equality, we need to cancel $\mathcal{P}$. Notice that $\mathcal{P}^2 = 1$. Thus, we assume by induction that $P^{-1}A^{\alpha}P = \mathcal{P}^{\alpha}_{\ \sigma}A^{\sigma}$. We proceed to prove this:

$$e\mathcal{P}^{\mu}_{\ \nu}\overline{\Psi}\gamma^{\nu}\Psi\mathcal{P}^{\alpha}_{\ \sigma}A^{\sigma}g_{\alpha\mu} \stackrel{?}{=} e\overline{\Psi}\gamma^{\mu}\Psi A_{\mu}$$

This is obviously true if $\mathcal{P}^{\mu}_{\ \nu}\mathcal{P}^{\alpha}_{\ \sigma}g_{\alpha\mu} = g_{\nu\sigma}$. We have:

$$\mathcal{P}^{\mu}_{\ \nu}\mathcal{P}^{\alpha}_{\ \sigma}g_{\alpha\mu} = \mathcal{P}^{\mu}_{\ \nu}\mathcal{P}_{\mu\sigma} = -\delta_{\mu\sigma}\mathcal{P}^{\mu}_{\ \nu} = g_{\nu\sigma}$$

Where the first equality is just index gymnastics; the second equality is because $\mathcal{P}_{\alpha\beta} = -\delta_{\alpha\beta}$ (multiply the matrices explicitely if you don't see this), and the third equality is because the negative $\mathcal{P}$ matrix is the metric.

Thus, we have:

$$\boxed{P^{-1}A^{\alpha}P = \mathcal{P}^{\alpha}_{\ \sigma}A^{\sigma}}$$

For T, the same argument holds: equation 40.37 introduces an additional negative sign, but this is cancelled because $T_{\alpha\beta} = g_{\alpha\beta}$, without a negative sign. Thus:

$$\boxed{T^{-1}A^{\alpha}T = \mathcal{T}^{\alpha}_{\ \sigma}A^{\sigma}}$$

Finally for C, the situation is even easier: equation 40.37 introduces a negative sign only (no matrix), and so the conjugation of A^μ must only restore the negative sign. Thus:

$$\boxed{C^{-1}A^\mu C = -A^\mu}$$

Srednicki 58.2. Furry's Theorem. Show that any scattering amplitude with no external fermions and an odd number of external photons is zero.

Though it's a little unclear from the problem statement, we will assume that this problem is specific to spinor electrodynamics, and so external fermions and photons are the only option. Since there are no external fermions, the correlation function must look like this:

$$\langle 0|A^\mu(x)A^\nu(y)\ldots|0\rangle$$

Using our result from the previous problem, let's insert a bunch of identities:

$$\langle 0|CC^{-1}A^\mu(x)CC^{-1}A^\nu(y)CC^{-1}\ldots CC^{-1}|0\rangle$$

Charge conjugating a vacuum won't do much, and so:

$$\langle 0|C^{-1}A^\mu(x)CC^{-1}A^\nu(y)CC^{-1}\ldots C|0\rangle$$

Now we use the result of the previous problem:

$$(-1)^n\langle 0|A^\mu(x)A^\nu(y)\ldots|0\rangle$$

If n is even, everything is fine. If n is odd, this will vanish, and so will the amplitude, according to the LSZ formula.