Srednicki Chapter 56

QFT Problems & Solutions

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Srednicki 56.1. Use equations 55.11 and 55.21-55.23 to verify equations 56.9 and 56.10.

Equations 56.9 and 56.10 together give the following, which we have to verify:

$$\langle 0|TA^i(x)A^j(y)|0\rangle \stackrel{?}{=} \frac{1}{i}\int \frac{d^4k}{(2\pi)^4}\frac{e^{ik(x-y)}}{k^2-i\varepsilon}\sum_{\lambda=\pm}\varepsilon_\lambda^{i*}(\vec{k})\varepsilon_\lambda^j(\vec{k})$$

Let's take this left hand side, and use equation 55.11:

$$\langle 0|TA^i(x)A^j(y)|0\rangle = \langle 0|T\sum_{\lambda=\pm}\int \widetilde{dk}\left[\varepsilon_\lambda^{i*}(\vec{k})a_\lambda(\vec{k})e^{ikx} + \varepsilon_\lambda^i(\vec{k})a_\lambda^\dagger(\vec{k})e^{-ikx}\right]$$

$$\sum_{\lambda'=\pm}\int \widetilde{dk'}\left[\varepsilon_{\lambda'}^{j*}(\vec{k}')a_{\lambda'}(\vec{k}')e^{ik'y} + \varepsilon_{\lambda'}^j(\vec{k}')a_{\lambda'}^\dagger(\vec{k}')e^{-ik'y}\right]|0\rangle$$

We can multiply. Two of these will immediately vanish; the remaining two give:

$$\langle 0|TA^i(x)A^j(y)|0\rangle = \sum_{\lambda=\pm}\sum_{\lambda'=\pm}\int \widetilde{dk}\widetilde{dk'}\Big\{\langle 0|T\varepsilon_\lambda^{i*}(\vec{k})a_\lambda(\vec{k})e^{ikx}\varepsilon_{\lambda'}^j(\vec{k}')a_{\lambda'}^\dagger(\vec{k}')e^{-ik'y}|0\rangle$$

$$+ \langle 0|T\varepsilon_\lambda^i(\vec{k})a_\lambda^\dagger(\vec{k})e^{-ikx}\varepsilon_{\lambda'}^{j*}(\vec{k}')a_{\lambda'}(\vec{k}')e^{ik'y}|0\rangle\Big\}$$

This gives:

$$\langle 0|TA^i(x)A^j(y)|0\rangle = \sum_{\lambda=\pm}\sum_{\lambda'=\pm}\int \widetilde{dk}\widetilde{dk'}\varepsilon_\lambda^{i*}(\vec{k})e^{i(kx-k'y)}\varepsilon_{\lambda'}^j(\vec{k}')\langle 0|Ta_\lambda(\vec{k})a_{\lambda'}^\dagger(\vec{k}')|0\rangle$$

Now we use equation 55.23. The term with operators vanishes; the constant term remains:

$$\langle 0|TA^i(x)A^j(y)|0\rangle = \sum_{\lambda=\pm}\sum_{\lambda'=\pm}\int \widetilde{dk}\widetilde{dk'}\varepsilon_\lambda^{i*}(\vec{k})e^{i(kx-k'y)}\varepsilon_{\lambda'}^j(\vec{k}')\langle 0|(2\pi)^3 2\omega\delta^3(\vec{k}'-\vec{k})\delta_{\lambda\lambda'}|0\rangle$$

This will allow us to do the k' integral and the sum over λ':

$$\langle 0|TA^i(x)A^j(y)|0\rangle = \sum_{\lambda=\pm}\int \widetilde{dk}\varepsilon_\lambda^{i*}(\vec{k})e^{ik(x-y)}\varepsilon_\lambda^j(\vec{k})\langle 0|0\rangle$$

which we can write as:

$$\langle 0|TA^i(x)A^j(y)|0\rangle = \int \widetilde{dk}\; e^{ik(x-y)} \sum_{\lambda=\pm} \varepsilon_\lambda^{i*}(\vec{k})\varepsilon_\lambda^j(\vec{k})$$

Now we can match up equation 8.11 and 8.13 to write this as:

$$\langle 0|TA^i(x)A^j(y)|0\rangle \stackrel{\checkmark}{=} \frac{1}{i}\int \frac{d^4k}{(2\pi)^4}\frac{e^{ik(x-y)}}{k^2 - i\varepsilon} \sum_{\lambda=\pm} \varepsilon_\lambda^{i*}(\vec{k})\varepsilon_\lambda^j(\vec{k})$$

as expected.