Srednicki Chapter 5

QFT Problems & Solutions

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Srednicki 5.1. Work out the LSZ reduction formula for the complex scalar field that was introduced in problem 3.5. Note that we must specify the type (a or b) of each incoming and outgoing particle.

The analog of 5.14 is:

$$\langle f|i\rangle = \sum_{i,j,i',j'} \langle 0|Ta_{i'}(+\infty)b_{j'}(+\infty)a_i^\dagger(-\infty)b_j^\dagger(-\infty)|0\rangle \tag{5.1.1}$$

In problem 3.5, we worked out some of these:

$$\begin{aligned} a(\mathbf{k}) &= \textstyle\int d^3x e^{-ikx}(\omega\phi(x) + i\partial_0\phi(x)) \\ b(\mathbf{k}) &= \textstyle\int d^3x e^{-ikx}(\omega\phi^\dagger(x) + i\partial_0\phi^\dagger(x)) \end{aligned} \tag{5.1.2}$$

Taking the Hermitian conjugate:

$$\begin{aligned} a^\dagger(\mathbf{k}) &= \textstyle\int d^3x e^{ikx}(\omega\phi^\dagger(x) - i\partial_0\phi^\dagger(x)) \\ b^\dagger(\mathbf{k}) &= \textstyle\int d^3x e^{ikx}(\omega\phi(x) - i\partial_0\phi(x)) \end{aligned} \tag{5.1.3}$$

Now the question becomes, does equation 5.10 (and therefore 5.11 and 5.12) hold also for b? Looking at (5.1.2) and (5.1.3), we see that when we switch to b, we have to switch to $\phi^\dagger$. Other than that, the derivation is still good.

As a result, equation 5.15 is still good, but now there are four types of terms, and those ϕ terms corresponding to the b operator become $\phi^\dagger$ terms. Let's write this out in full:

$$\langle f|i\rangle = i^{i+j+i'+j'} \int d^4x_i e^{ik_ix_i}(-\partial_i^2+m^2)\dots d^4x_j e^{ik_jx_j}(-\partial_j^2+m^2)\dots d^4x_{i'}e^{ik_{i'}x_{i'}}(-\partial_{i'}^2+m^2)\dots$$

$$d^4x_{j'}e^{ik_{j'}x_{j'}}(-\partial_{j'}^2+m^2)\dots \times \langle 0|T\phi(x_i)\dots\phi^\dagger(x_j)\dots\phi(x_{i'})\dots\phi^\dagger(x_{j'})\dots|0\rangle$$