Srednicki Chapter 45

QFT Problems & Solutions

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Srednicki 45.1. (a) Determine how $\phi(x)$ must transform under parity, time reversal, and charge conjugation in order for these to all be symmetries of the theory.

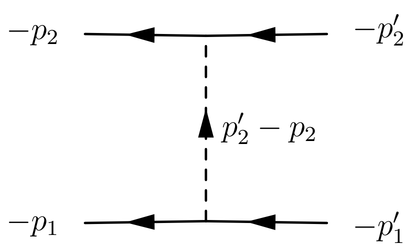
Equations 40.37, 40.41, and 40.47 show that $\overline{\Psi}\Psi$ is even under parity, time reversal, and charge conjugation. It follows that $\phi(x)$ must also be even under parity, time reversal, and charge conjugation if $g\phi\overline{\Psi}\Psi$ is to be even (invariant) under them.

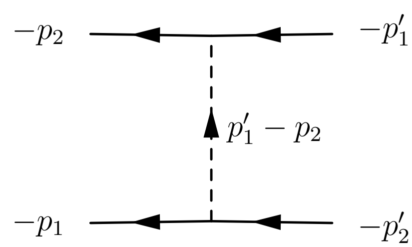
(b) Same question, but this time with the interaction given by $\mathcal{L}_1 = ig\phi\overline{\Psi}\gamma_5\Psi$ instead of equation 45.1.

Same logic, but this time $\phi(x)$ must be even under charge conjugation and odd under parity and time reversal.

Srednicki 45.2. Use the Feynman rules to write down at tree level $i\mathcal{T}$ for the processes $e^+e^+ \to e^+e^+$ and $\phi\phi \to e^+e^-$.

We draw the diagrams according to the Feynman Rules presented in the chapter:





Note that the direction on the internal scalar is arbitrary. Next we follow the Feynman rules, starting at the end of the fermionic lines and write down the values of each line:

$$\overline{v}_{s_2}(p_2)(ig)\frac{(-i)}{k^2 + M^2 - i\varepsilon}v_{s_2'}(p_2')\overline{v}_{s_1}(p_1)(ig)v_{s_1'}(p_1')$$

and

$$\overline{v}_{s_2}(p_2)(ig)\frac{(-i)}{k^2 + M^2 - i\varepsilon}v_{s_1'}(p_1')\overline{v}_{s_1}(p_1)(ig)v_{s_2'}(p_2')$$

The two fermion lines have are the same on the left but opposite on the right, so the overall phase is negative. It doesn't matter which diagram gets which sign; all we care about is the relative sign. Thus:

$$\begin{aligned} i\mathcal{T} &= \overline{v}_{s_2}(p_2)(ig)\frac{(-i)}{(p_2'-p_2)^2+M^2-i\varepsilon}v_{s_2'}(p_2')\overline{v}_{s_1}(p_1)(ig)v_{s_1'}(p_1') \\ &-\overline{v}_{s_2}(p_2)(ig)\frac{(-i)}{(p_1'-p_2)^2+M^2-i\varepsilon}v_{s_1'}(p_1')\overline{v}_{s_1}(p_1)(ig)v_{s_2'}(p_2') \end{aligned}$$

Simplifying:

$$i\mathcal{T} = ig^2\left[\frac{\overline{v}_{s_2}(p_2)v_{s_2'}(p_2')\overline{v}_{s_1}(p_1)v_{s_1'}(p_1')}{(p_2'-p_2)^2+M^2-i\varepsilon} - \frac{\overline{v}_{s_2}(p_2)v_{s_1'}(p_1')\overline{v}_{s_1}(p_1)v_{s_2'}(p_2')}{(p_1'-p_2)^2+M^2-i\varepsilon}\right]$$

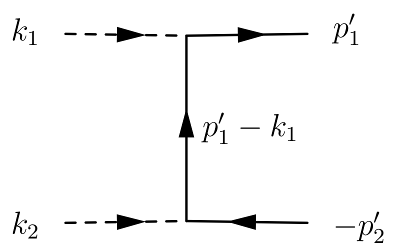
For any physically allowed value of the denominator, the denominator will not vanish. We can therefore neglect the infinitesimal:

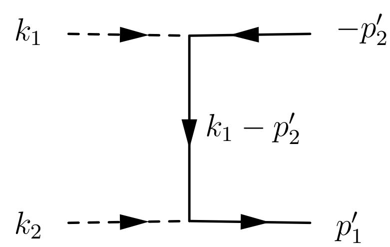
$$i\mathcal{T} = ig^2\left[\frac{\overline{v}_{s_2}(p_2)v_{s_2'}(p_2')\overline{v}_{s_1}(p_1)v_{s_1'}(p_1')}{(p_2'-p_2)^2+M^2} - \frac{\overline{v}_{s_2}(p_2)v_{s_1'}(p_1')\overline{v}_{s_1}(p_1)v_{s_2'}(p_2')}{(p_1'-p_2)^2+M^2}\right]$$

Notice that these are the Mandelstam variables:

$$\boxed{i\mathcal{T} = ig^2\left[\frac{\overline{v}_{s_2}(p_2)v_{s_2'}(p_2')\overline{v}_{s_1}(p_1)v_{s_1'}(p_1')}{-t+M^2} - \frac{\overline{v}_{s_2}(p_2)v_{s_1'}(p_1')\overline{v}_{s_1}(p_1)v_{s_2'}(p_2')}{-u+M^2}\right]}$$

Now for $\phi\phi \to e^+e^-$:





Following the Feynman rules, we start at the end of the fermionic lines and write down the values of each line:

$$\overline{u}_{s_1'}(p_1')(ig)(1)\frac{-i(-\not{p}+m)}{p^2+m^2-i\varepsilon}(ig)(1)\overline{v}_{s_2'}(p_2')$$

For the first diagram, we have $p = p_1' - k_1$; for the second, it is $p = k_1 - p_2'$. In both diagrams there is only one fermionic line, so the relative sign is positive. Then:

$$i\mathcal{T} = \overline{u}_{s_1'}(p_1')(ig)(1)\frac{-i(-\not{p}_1'+\not{k}_1+m)}{(p_1'-k_1)^2+m^2-i\varepsilon}(ig)(1)\overline{v}_{s_2'}(p_2') + \overline{u}_{s_1'}(p_1')(ig)(1)\frac{-i(-\not{k}_1+\not{p}_2'+m)}{(k_1-p_2')^2+m^2-i\varepsilon}(ig)(1)\overline{v}_{s_2'}(p_2')$$

We simplify:

$$i\mathcal{T} = ig^2\left[\overline{u}_{s_1'}(p_1')\frac{(-\not{p}_1'+\not{k}_1+m)}{(p_1'-k_1)^2+m^2-i\varepsilon}\overline{v}_{s_2'}(p_2') + \overline{u}_{s_1'}(p_1')\frac{(-\not{k}_1+\not{p}_2'+m)}{(k_1-p_2')^2+m^2-i\varepsilon}\overline{v}_{s_2'}(p_2')\right]$$

This gives:

$$i\mathcal{T} = ig^2 \overline{u}_{s_1'}(p_1') \left[\frac{(-\not{p}_1' + \not{k}_1 + m)}{(p_1' - k_1)^2 + m^2 - i\varepsilon} + \frac{(-\not{k}_1 + \not{p}_2' + m)}{(k_1 - p_2')^2 + m^2 - i\varepsilon} \right] \overline{v}_{s_2'}(p_2')$$

Notice that we have Mandelstam variables in the denominator:

$$i\mathcal{T} = ig^2 \overline{u}_{s_1'}(p_1') \left[\frac{(-\not{p}_1' + \not{k}_1 + m)}{-t + m^2 - i\varepsilon} + \frac{(-\not{k}_1 + \not{p}_2' + m)}{-u + m^2 - i\varepsilon} \right] \overline{v}_{s_2'}(p_2')$$

As before, we drop the infinitesimal:

$$\boxed{i\mathcal{T} = ig^2 \overline{u}_{s_1'}(p_1') \left[\frac{-\not{p}_1' + \not{k}_1 + m}{-t + m^2} + \frac{\not{p}_2' - \not{k}_1 + m}{-u + m^2} \right] \overline{v}_{s_2'}(p_2')}$$

Note: Recall that the internal particle in Yukawa Theory is a scalar; the more important case, a photon, will be handled later (the complication is that the photon is spin 1).