Srednicki Chapter 42

QFT Problems & Solutions

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Srednicki 42.1. Prove equation 42.24 directly, using properties of the $\mathcal{C}$ matrix.

Using equation 42.12, we have:

$$[S(x-y)\mathcal{C}^{-1}]_{\alpha\beta} = \int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2+m^2-i\varepsilon}\left[(-\not{p}+m)\mathcal{C}\right]_{\alpha\beta} \tag{42.1.1}$$

Now we consider the part in brackets:

$$\left[(-p_\mu\gamma^\mu+m)\mathcal{C}^{-1}\right]^T = \mathcal{C}^{-1T}(-p_\mu\gamma^\mu+m)^T$$

Recall that $C^{-1}=C^T$. Thus:

$$\left[(-p_\mu\gamma^\mu+m)\mathcal{C}^{-1}\right]^T = \mathcal{C}[-p_\mu(\gamma^\mu)^T+m]$$

Using equation 36.40, we have:

$$\left[(-p_\mu\gamma^\mu+m)\mathcal{C}^{-1}\right]^T = \mathcal{C}[p_\mu\mathcal{C}^{-1}(\gamma^\mu)\mathcal{C}+m]$$

which gives:

$$\left[(-p_\mu\gamma^\mu+m)\mathcal{C}^{-1}\right]^T = (\not{p}+m)\mathcal{C}$$

Using this, we can rewrite equation (42.1.1) as:

$$[S(x-y)\mathcal{C}^{-1}]_{\alpha\beta} = \int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2+m^2-i\varepsilon}\left[(\not{p}+m)\mathcal{C}\right]_{\beta\alpha}$$

Since $\mathcal{C}=-\mathcal{C}^{-1}$, we have:

$$[S(x-y)\mathcal{C}^{-1}]_{\alpha\beta} = -\int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2+m^2-i\varepsilon}\left[(\not{p}+m)\mathcal{C}\right]_{\beta\alpha}$$

We'd like to use equation 42.12 again, but we need $\not{p}$ to be negative. Therefore we simply switch out integration variable $p\to -p$:

$$[S(x-y)\mathcal{C}^{-1}]_{\alpha\beta} = -\int \frac{d^4p}{(2\pi)^4}\frac{e^{-ip(x-y)}}{p^2+m^2-i\varepsilon}\left[(-\not{p}+m)\mathcal{C}\right]_{\beta\alpha}$$

Now the problem is the negative sign in the exponential. We absorb that into the x and y terms:

$$[S(x-y)\mathcal{C}^{-1}]_{\alpha\beta} = -\int \frac{d^4p}{(2\pi)^4} \frac{e^{ip(y-x)}}{p^2+m^2-i\varepsilon} [(-\not{p}+m)\mathcal{C}]_{\beta\alpha}$$

which allows us to use equation 42.12:

$$[S(x-y)\mathcal{C}^{-1}]_{\alpha\beta} = -[S(y-x)\mathcal{C}^{-1}]_{\beta\alpha}$$

as expected.