Srednicki Chapter 41

QFT Problems & Solutions

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Srednicki 41.1. Assuming that equation 39.40 holds for the exact single-particle states, verify equations 41.24 and 41.25, up to overall scale.

It's a little bit unclear what we're supposed to do. Equation 39.40 by itself won't get us very far. Srednicki uses equation 39.41 in his solution: this has the advantage of showing the Lorentz Invariance explicitely, but seems unnecessarily complicated.

I'll prove each equation in a different method. First, let's prove 41.24 by considering a modified version of Srednicki's solution. We have:

$$\langle p, s, -|\Psi(x)|0\rangle = \langle p, s, -|U(\Lambda)U(\Lambda)^{-1}\Psi(x)U(\Lambda)U(\Lambda)^{-1}|0\rangle$$

The right most unitary operator acts on the vacuum, where it won't do much. We can use two of the remaining unitary operators to transform $\Psi(x)$:

$$\langle p, s, -|\Psi(x)|0\rangle = \langle p, s, -|U(\Lambda)\Psi(\Lambda^{-1}x)|0\rangle$$

Now we use equation 39.41:

$$\langle p, s, -|\Psi(x)|0\rangle = \sum_{s'} R^*_{ss'}(\Lambda^{-1}, \vec{p})\langle \Lambda^{-1}p, s, -|\Psi(\Lambda^{-1}x)|0\rangle$$

Now we're supposed to verify 41.24, so let's use that now. *Note the logic here. This is essentially a proof by contradiction: every other step in the problem is true, so if our result is false, it proves that this assumption was wrong. Our result will be true, which suggests that the assumption is true (in fact, it proves that the assumption is true so long as we don't make any irreversible steps).*

$$v_s(\vec{p})e^{-ipx} = \sum_{s'} R^*_{ss'}(\Lambda^{-1}, \vec{p})v_{s'}(\Lambda^{-1}\vec{p})e^{-ipx}$$

where the exponential has a product of two four vectors, and is therefore invariant under the Lorentz Transformation. We can therefore kill the exponentials:

$$v_s(\vec{p}) = \sum_{s'} R^*_{ss'}(\Lambda^{-1}, \vec{p})v_{s'}(\Lambda^{-1}\vec{p})$$

Now remember that $R_{ss'}$ is a product of $D(\Lambda)$ sandwiched between some spinors. The result is a number, and so the equation is true up to overall scale, as expected. $\square$

It would be trivial to use this method to prove equation 41.25, but let's instead use what I consider to be a better proof. We have, from the mode expansion:

$$\langle p, s, +|\overline{\Psi}(x)|0\rangle = \sum_{s'=\pm} \int \widetilde{dp} \langle p, s, -|b_s^\dagger(\vec{p'})\overline{u}_s(\vec{p'})e^{-ip'x} + d_s(\vec{p'})\overline{v}_s(\vec{p'})e^{ip'x}|0\rangle$$

The annihilation and creation operators are the only thing that can act on the vacuum. Further, the annihilation operator acting on the vacuum will give zero. Thus:

$$\langle p, s, +|\overline{\Psi}(x)|0\rangle = \sum_{s'=\pm} \int \widetilde{dp'}\ \overline{u}_s(\vec{p'})e^{-ip'x}\langle p, s, +|b_s^\dagger(\vec{p'})|0\rangle$$

Now we use equation 41.1:

$$\langle p, s, +|\overline{\Psi}(x)|0\rangle = \sum_{s'=\pm} \int \widetilde{dp'}\ \overline{u}_s(\vec{p'})e^{-ip'x}\langle p, s, +|p', s', +\rangle$$

Now we use equation 41.5: *Note that this, coupled with the operator definition, is the key to the problem. This is the normalization that we need to enforce, even at the cost of renormalizing.*

$$\langle p, s, +|\overline{\Psi}(x)|0\rangle = \sum_{s'=\pm} \int \widetilde{dp}\ \overline{u}_s(\vec{p})e^{-ip'x}(2\pi)^3 2\omega\delta_{ss'}\delta^3(\vec{p} - \vec{p'})$$

which is:

$$\langle p, s, +|\overline{\Psi}(x)|0\rangle = \overline{u}_s(\vec{p})e^{-ipx}$$

as expected. $\square$